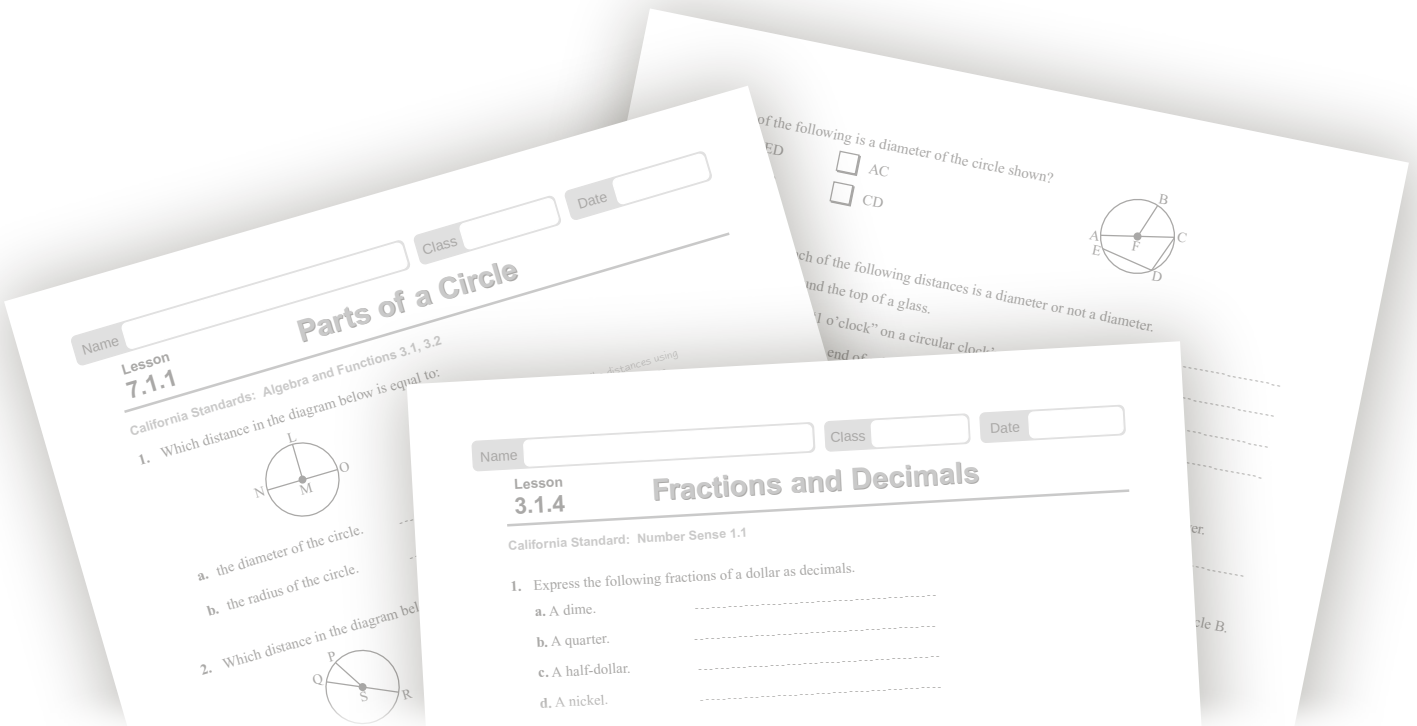
CGP Education

California
Mathematics
Course One



Homework Book
California Standards-Driven Program

California

Mathematics

Course One

Homework Book

California Standards-Driven Program

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This book covers each of the requirements of the California grade 6 Standards.

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California Grade Six Mathematics Standards

The following table lists all the California Mathematics Content Standards for grade 6, with cross references to where each Standard is covered in this Program. This will enable you to measure your progression against the California Grade 6 Standards, as you work your way through the Program.

California Standard	Number Sense	
1.0	★ Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:	**Chapters 1, 3, 4**
1.1	★ Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.	**Chapters 1, 3**
1.2	★ Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (a/b, a to b, $a:b$).	**Chapter 4**
1.3	★ Use proportions to solve problems (e.g., determine the value of N if $\frac{4}{7}=\frac{N}{21}$, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.	**Chapter 4**
1.4	★ Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.	**Chapter 3**
2.0	★ Students calculate and solve problems involving addition, subtraction, multiplication, and division:	**Chapters 1, 3**
2.1	Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.	**Chapter 3**
2.2	Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., $\frac{5}{8}\div\frac{15}{16}=\frac{5}{8}\times\frac{16}{15}=\frac{2}{3}$).	**Chapter 3**
2.3	★ Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.	**Chapter 1**
2.4	★ Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).	**Chapter 3**

California Standard	Algebra and Functions	
1.0	Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:	**Chapter 2**
1.1	★ Write and solve one-step linear equations in one variable.	**Chapter 2**
1.2	Write and evaluate an algebraic expression for a given situation, using up to three variables.	**Chapter 2**
1.3	Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process.	**Chapter 2**
1.4	Solve problems manually by using the correct order of operations or by using a scientific calculator.	**Chapter 2**
2.0	Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:	**Chapter 4**
2.1	Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).	**Chapter 4**

2.2	★	Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.	**Chapter 4**
2.3		Solve problems involving rates, average speed, distance, and time.	**Chapter 4**
3.0		Students investigate geometric patterns and describe them algebraically:	**Chapter 2, 7**
3.1		Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$, $A = \frac{1}{2}bh$, $C = \pi d$ — the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).	**Chapter 2, 7**
3.2		Express in symbolic form simple relationships arising from geometry.	**Chapter 2, 7**

California Standard

Measurement and Geometry

1.0		Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:	**Chapter 7**
1.1	★	Understand the concept of a constant such as π; know the formulas for the circumference and area of a circle.	**Chapter 7**
1.2		Know common estimates of π (3.14; $\frac{22}{7}$) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.	**Chapter 7**
1.3		Know and use the formulas for the volume of triangular prisms and cylinders (area of base × height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.	**Chapter 7**
2.0		Students identify and describe the properties of two-dimensional figures:	**Chapter 7**
2.1		Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.	**Chapter 7**
2.2	★	Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.	**Chapter 7**
2.3		Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).	**Chapter 7**

California Standard

Statistics, Data Analysis and Probability

1.0		Students compute and analyze statistical measurements for data sets:	**Chapter 5**
1.1		Compute the range, mean, median, and mode of data sets.	**Chapter 5**
1.2		Understand how additional data added to data sets may affect these computations.	**Chapter 5**
1.3		Understand how the inclusion or exclusion of outliers affects these computations.	**Chapter 5**
1.4		Know why a specific measure of central tendency (mean, median) provides the most useful information in a given context.	**Chapter 5**
2.0		Students use data samples of a population and describe the characteristics and limitations of the samples:	**Chapter 5**
2.1		Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.	**Chapter 5**
2.2	★	Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.	**Chapter 5**
2.3	★	Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.	**Chapter 5**

California Grade Six Mathematics Standards

2.4	★ Identify data that represent sampling errors and explain why the sample (and the display) might be biased.	**Chapter 5**
2.5	★ Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.	**Chapter 5**
3.0	Students determine theoretical and experimental probabilities and use these to make predictions about events:	**Chapter 6**
3.1	★ Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.	**Chapter 6**
3.2	Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).	**Chapter 6**
3.3	★ Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, $1 - P$ is the probability of an event not occurring.	**Chapter 6**
3.4	Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.	**Chapter 6**
3.5	★ Understand the difference between independent and dependent events.	**Chapter 6**

Mathematical Reasoning

California Standard		
1.0	Students make decisions about how to approach problems:	**Illustrated in Chapters 1, 2, 4, and 7, and throughout Program**
1.1	Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.	
1.2	Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.	
1.3	Determine when and how to break a problem into simpler parts.	
2.0	Students use strategies, skills, and concepts in finding solutions:	
2.1	Use estimation to verify the reasonableness of calculated results.	
2.2	Apply strategies and results from simpler problems to more complex problems.	
2.3	Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.	
2.4	Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.	
2.5	Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.	
2.6	Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.	
2.7	Make precise calculations and check the validity of the results from the context of the problem.	
3.0	Students move beyond a particular problem by generalizing to other situations:	
3.1	Evaluate the reasonableness of the solution in the context of the original situation.	
3.2	Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.	
3.3	Develop generalizations of the results obtained and the strategies used and apply them in new problem situations.	

★ Shows **key standards**

Advice for Parents and Guardians

What the Homework Book is For

Homework helps students improve their thinking skills and develop learning outside the classroom. This Homework Book is an integral part of the CGP California Standards-Driven Course One Program.

- It focuses purely on the California Content Standards for grade 6 with no extraneous content.
- It has been written to match the California Grade 6 Content Standards, using the Mathematics Framework for California Public Schools (2005) as a guide. The Standards are listed on pages iv–vi of this book.
- It has a clear and simple structure, which is the same in each component of the Program.
- It is a flexible program which caters for a diverse student body.

This Homework Book follows exactly the same structure and order of teaching as the Student Textbook.

- The Course One Program is broken down into seven Chapters (see pages ix–xi for more detail).
- These Chapters are in turn divided into smaller Sections, which cover broad areas of the Course One program.
- These Sections are then broken down into smaller, manageable Lessons, which are designed to be worked through in a typical 50-minute math class.
- Each Lesson in the Student Textbook starts with the relevant California standard in full. This is then linked to a clear learning objective written in everyday language so that your child can understand what the Lesson is about and how it fits in with the overall California Grade 6 Standards.

At the end of each Lesson, the teacher will assign homework from this book, which contains one worksheet for each Lesson of the Course One Program. That means that there is always additional work for your child to practice the skills learned in the math Lesson.

Each worksheet is perforated and hole punched — so your child can hand each sheet in to the teacher, and then store the marked and corrected sheets in a separate file at home or at school.

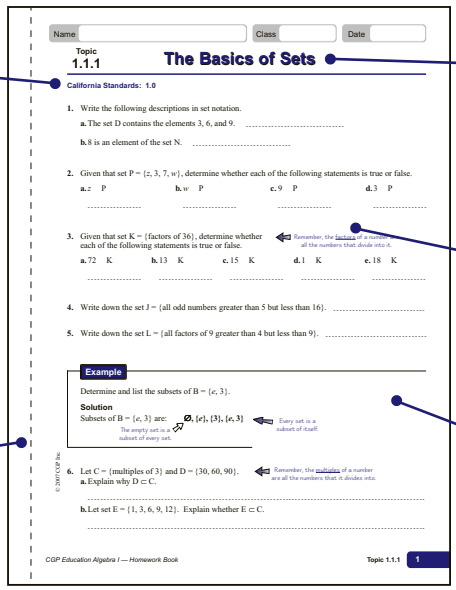
Using the Homework Book

The worksheets in this book have been designed to be straightforward to use.
The worksheets have lots of common features:

The relevant California Mathematics Standard is always stated at the start of each homework sheet.

Each homework sheet covers several difficulty levels. The teacher has information about which problems are suitable for each student so will set specific problems for your child.

The pages are perforated so that each homework sheet can be pulled out and handed in to the teacher.


Name Class Date

Topic 1.1.1 The Basics of Sets

California Standards: 1.0

1. Write the following descriptions in set notation.
a. The set D contains the elements 3, 6, and 9.
b. 8 is an element of the set N.

2. Given that set P = {z, 3, 7, w}, determine whether each of the following statements is true or false.
a. z P b. w P c. 9 P d. 3 P

3. Given that set K = {factors of 36}, determine whether each of the following statements is true or false.
Remember, the factors of a number are all the numbers that divide into it.
a. 72 K b. 13 K c. 15 K d. 1 K e. 18 K

4. Write down the set J = {all odd numbers greater than 5 but less than 16}.

5. Write down the set L = {all factors of 9 greater than 4 but less than 9}.

Example

Determine and list the subsets of B = {e, 3}.

Solution
Subsets of B = {e, 3} are: Ø, {e}, {3}, {e, 3}
The empty set is a subset of every set.
Every set is a subset of itself.

6. Let C = {multiples of 3} and D = {30, 60, 90}.
Remember, the multiples of a number are all the numbers that it divides into.
a. Explain why D ⊂ C.
b. Let E = {1, 3, 6, 9, 12}. Explain whether E ⊂ C.

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CGP Education Algebra I — Homework Book Topic 1.1.1 1

There's one homework sheet for every Lesson in the book — so it's easy to refer back to the relevant part of the Textbook.

Worked examples help students to answer problems and show them how to write their own solutions.

Useful hints help students to understand each problem.

At the back of the book you will find worked solutions to several problems from each worksheet. Together with the worked example on the worksheet itself, this will allow you to check that your child understands the key concepts for each worksheet.

This book contains more than just questions. Pages ix–xi give Chapter Overviews with lists of quick-reference Key Notation and Terminology, and pages xii–xiii contain advice on Question Technique.

Advice for Parents and Guardians

By getting actively involved in your child's education, you can make a real difference to his or her success.

Even if you are less confident with the math yourself, you can still provide help — both on a practical level and in less direct ways. Just by showing an interest in your child's work, you can help improve their perception of the value of math.

Here is some practical, day-to-day advice on how to help your child get the most out of their Course One program.

Provide a Suitable Working Environment

It is important that your child has an appropriate place to work in. It is very easy to get distracted at home, so each of the measures suggested below is designed to keep your child as focused as possible.

- He or she should work at an uncluttered table or desk — a kitchen table is fine for this.
- The environment should contain as few distractions as possible — for example, if there is a TV in the room, make sure it is turned off and your child is facing away from it (even a turned off TV can be distracting).
- Try to set aside a regular time each day for working. This can be difficult to fit around other commitments, but it is worth making the effort. Having a homework slot as part of your family's regular schedule can help your child to get into the right frame of mind to work.
- Encourage your child to work solidly for twenty minutes, then take a five minute break. Even with the best intentions, many children find it hard to focus for long periods of time — so working in short, intensive bursts is often most effective. The short break can also be used as an incentive — working hard for twenty minutes straight earns five minutes off.

Try to Identify Problem Areas

If your child is struggling with a piece of work, there are several possible root causes. Try to identify which aspects of a question your child is having difficulty with, so that he or she can take steps to solve the problem.

- Ask your child to explain in words how the problem would be solved. Struggling with this may suggest that the basic concept behind the question has not been grasped. (You do not have to understand the math yourself to do this, you just need to judge whether your child can explain the concept clearly.) See page xii, Concept Questions. It is useful to go through this process even if your child is doing well. Students often learn procedures for answering questions without understanding the underlying math. Although this can be sufficient for simple problems, they may run into difficulties later if earlier concepts are missed.
- If your child understands the concept, but still cannot answer the question correctly, the problem may be with a particular component skill (for example, one step in the work). Read through a worked example with your child (you will find these in every Lesson of the Homework Book). Then copy out the question onto a separate piece of paper and ask your child to try to answer it. Compare each step of the work with that given in the book to see where the problem is.
- The difficulty may lie with the type of question. On pages xii and xiii, Guidance on Question Technique, there are lists of measures that can be taken to deal with particular question types.

Keep in Contact with the School

You ought to receive regular reports on your child's progress from the school, but remember that communication between home and school can be in two directions. If you are concerned about any aspect of your child's progress, it may help to discuss the issue with his or her teacher.

Overviews of Chapter Content

Course One is not a course that can be treated in isolation. In each Chapter, you learn new concepts that are part of the larger picture of mathematics, and everything you learn builds on your knowledge from previous grades.

Chapter One — Ordering and Manipulating Numbers

Chapter 1 is about integers and decimals. It also discusses rounding and estimation.

The following concepts are covered in this Chapter:

- natural numbers, whole numbers, and integers
- comparing numbers using a number line
- adding and subtracting integers and decimals, and what this looks like on the number line
- multiplying and dividing integers and decimals, and what this looks like on the number line
- rounding numbers (replacing one number with another number that's easier to work with)
- estimation ("guessing" the value of a number whose exact value you don't know)

How the Chapter follows on from previous study:

You have been working with integers and the basic operations of addition, subtraction, multiplication, and division since very early grades. This Chapter extends what you have learned previously.

Key Notation and Terminology:

Comparing Numbers

One number is less than another if it lies further to the left on a number line; "$<$" means "is less than."
One number is greater than another if it lies further to the right on a number line; "$>$" means "is greater than."

Numbers that are greater than zero are positive.
Numbers that are less than zero are negative.

Important Sets of Numbers

Natural numbers are 1, 2, 3, 4...
Whole numbers are 0, 1, 2, 3, 4...
Integers are 0, 1 and –1, 2 and –2, 3 and –3, 4 and –4...

Terms in Word Problems

sum:	the result of adding two expressions
difference:	the result of subtracting one expression from another
product:	the result of multiplying two expressions
quotient:	the result of dividing one expression by another

Chapter Two — Expressions and Equations

Chapter 2 is about writing expressions and solving equations.

The following concepts are covered in this Chapter:

- using variables to represent unknown quantities
- the correct order in which to do mathematical operations
- solving 1-step equations
- analyzing complicated problems

How the Chapter follows on from previous study:

In grade 4, you used symbols to represent unknown quantities. You also used parentheses to show which parts of an expression to evaluate first.

In the same grade, you used formulas to find various quantities. You also learned how to manipulate one of these formulas by adding equal amounts to both sides, or multiplying both sides by equal amounts.

In grade 5, you used variables to represent unknown quantities, and wrote mathematical expressions involving variables. You also evaluated these expressions by substituting actual values for the variables.

Key Notation and Terminology:

Order of Operations

parentheses: **(...)**

exponents: in the expression b^x, the exponent is x

PEMDAS: shows which parts of an expression should be evaluated first;
Parentheses, Exponents,
Multiplication/Division,
Addition/Subtraction

Properties of Numbers

associative properties:	$(x + y) + z = x + (y + z)$ $(xy)z = x(yz)$
commutative properties:	$x + y = y + x$ $xy = yx$
distributive property:	$x(y + z) = xy + xz$

Algebra

variable:	a letter or symbol used to represent an unknown quantity
expression:	a collection of numbers, variables, and symbols that represent a quantity
equation:	a mathematical statement showing that two quantities are equal

Overviews of Chapter Content

Chapter Three — Fractions and Percentages

Chapter 3 is about fractions and percents.

The following concepts are covered in this Chapter:

- multiplying, dividing, adding, and subtracting fractions and mixed numbers
- simplifying fractions
- converting percents to fractions, and fractions to percents

How the Chapter follows on from previous study:

In grades 3 and 4, you saw what fractions are, and how you can add and subtract them if the denominators are the same.

In grade 5, you worked with decimals, fractions, and mixed numbers, and placed these on a number line. You also did some multiplication and division involving fractions.

Key Notation and Terminology:

Working with Fractions

denominator:	the bottom line of a fraction
numerator:	the top line of a fraction
proper fraction:	a fraction whose numerator is smaller than its denominator
improper fraction:	a fraction whose numerator is not smaller than its denominator
unit fraction:	a fraction with a numerator of 1
mixed number:	a number with a whole number part and a fraction part (like $2\frac{1}{2}$)
reciprocal:	the number you get by swapping over a fraction's numerator and denominator

Chapter Four — Ratio, Proportion, and Rate

Chapter 4 is about ratios and rates.

The following concepts are covered in this Chapter:

- ratios and rates
- proportions
- cross-multiplication
- scale drawings and maps

How the Chapter follows on from previous study:

Ratios can be written as a fraction, so some of this Chapter follows on from what you learned in Chapter 3.

In grade 3, you saw how to convert quantities from one set of units to another — for example, from meters to centimeters.

Key Notation and Terminology:

Ratios and Rates

ratios: these are ways to compare two quantities

proportions: equations that show that two ratios are equal

cross-multiplication: a method for solving proportions

rate: a special kind of ratio that has units

Geometry

similar: two shapes are similar if they are the same shape (they don't have to be the same size)

customary units:
units like inches, feet, yards, miles, and pounds

metric units:
units like meters, kilometers, and kilograms

Chapter Five — Data Sets

Chapter 5 is about statistics, and ways to analyze sets of data.

The following concepts are covered in this Chapter:

- describing a "typical value" for a set of data — using the mean, the median, and the mode
- describing how much numbers in a set vary, using the range
- using diagrams and graphs to display data
- methods that can be used to collect data

How the Chapter follows on from previous study:

In grade 4, you wrote survey questions that could be used to find out information from people. You also used graphs and charts to display your findings.

In grades 4 and 5, you looked at the mean, median, and mode, and saw that their values can be different.

Key Notation and Terminology:

Statistics

outliers:
values in a set of data that lie a long way from most of the other values

population:
the entire collection of objects or people you want to find out

sample:
a selection of objects from the population

Chapter Six — Probability

Chapter 6 is about probability.

The following concepts are covered in this Chapter:

- probability basics — actions, outcomes, and events
- calculating theoretical probabilities, using a list of outcomes
- calculating probabilities involving more than one event
- dependent and independent events
- experimental probability

How the Chapter follows on from previous study:

In grade 3, you saw what "probability" means in math — that it is a number between 0 and 1 that describes how likely or unlikely an event is to happen.

In grade 4, you saw how you can work out probabilities in some situations. To do this, you used tables, grids, and tree diagrams.

Key Notation and Terminology:

Important Probability Terms

outcome:
a possible result of an experiment

event:
a set of outcomes matching a particular condition

complement:
the complement of event A is the event "not A" — if event A does not happen, you can say that event "not A" has happened (and vice versa)

theoretical probability:
a probability based on thinking about an experiment

experimental probability:
a probability based on doing an experiment

Probability Notation

P(A)	the probability that event A will happen
P(not A)	the probability that event A will not happen

Chapter Seven — Geometry

Chapter 7 is about angles, shapes, and three-dimensional figures.

The following concepts are covered in this Chapter:

- circles, and how to find their area and circumference
- angles — including special pairs of angles
- triangles and quadrilaterals
- three-dimensional figures — including prisms, cubes, and cylinders

How the Chapter follows on from previous study:

In grades 4 and 5, you looked at how the area and perimeter of different shapes can be calculated.

In grade 4, you looked at triangles, and how triangles can be given different names depending on either their angles or the lengths of their sides. You also saw how circles can be described using their radius or diameter.

You studied the idea of volume in grade 5. You saw how you can find the volume of a box, and how the units of volume depended on the units used to measure the lengths of the sides.

Key Notation and Terminology:

General Terms

area: the size of a flat surface
volume: the amount of space inside a three-dimensional figure

Circles

radius: the distance from the center of a circle to its edge
diameter: a straight line from one side of a circle to the other, passing through the center

Types of Triangles

equilateral	all three sides the same length
isosceles	two sides the same length
scales	all three sides different lengths

Other Shapes

rectangle	a 4-sided shape with 4 right angles
parallelogram	a 4-sided shape with 2 pairs of parallel sides
rhombus	a parallelogram with 4 sides of equal length
trapezoid	a 4-sided shape with exactly 1 pair of parallel sides

Angles

right	90°, or a quarter-turn
acute	less than 90°
obtuse	greater than 90°

Three-Dimensional Figures

face	a flat surface on a three-dimensional figure
edge	where 2 faces meet
vertex (vertices)	where 3 or more faces meet; a "corner"

Guidance on Question Technique

There are a number of different abilities required in order to be successful in Course One — concepts have to be understood, skills have to be learned, and those skills should be applied to new situations. So math questions are not all the same.

This Course uses a Variety of Question Types

The five main types of question covered are:

- **Concept Questions**
- **Skills Practice Questions**
- **Applying Skills to New Situations**
- **Interesting/Challenging Questions**
- **Proof Questions**

In addition to this, many questions will be a mixture of types — for example, an application of learned skills to a new situation may also be an interesting mathematical problem.

It's Important to Understand These Differences

All of these types of questions can be tackled in different ways. If you struggle with one particular type, there are specific measures you can take, as outlined below.

Concept Questions

Concept Questions are those that help you to understand the topic, and help to check your understanding.

These may be probing questions asked by your teacher as part of the teaching process, such as: **"Dividing by ½ is the same as multiplying by which number? Why?"**

Or exercises that reinforce and check understanding, such as: **"State whether each of the triangles a) – f) is an equilateral, isosceles, or scalene triangle."**

Concept Questions are fundamental to the learning of mathematics, as they are based on understanding rather than skills. If you do not understand a concept, you may struggle to learn the necessary skills. At the same time, if you have a firm grasp of the concepts, the skills you learn will make much more sense.

If you are struggling with Concept Questions:

1) Go back to earlier work that you are comfortable with. This will give you a useful starting point.
2) Then gradually work through the concepts, one by one. Try looking at worked examples, making sure that you follow the reasoning behind every step.
3) This will give you a better understanding of where the math comes from than if you had merely "rote-learned" the facts.
4) If you can, return to the original questions that you were struggling with to check that you now understand the concepts.

Skills Practice Questions

These are drill-type questions that let you practice the skills you have just learned, and check that you have learned those skills properly, for example, **"For each of parts a) to v), use cross-multiplication to find the missing value."**

Skills Practice Questions are repetitive, and are designed this way to help you learn — it is generally easier to remember something that you've done 10 times, than something you've only done once.

These questions do not go beyond the scope of what you have learned in class; they simply provide lots of practice at using the same skills, over and over again.

If you are making mistakes in Skills Practice Questions, there are a few possible root problems:

1) It could be that you have not fully understood the concept, and so are not applying the method correctly. In this case, see the advice for Concept Questions.
2) If you do understand the concept, it may simply be that you need to brush up on one or more of the component skills. Try doing a worked example and comparing each of your steps with the steps in the book. That way you can see exactly where you are going wrong and get extra help with those topics if necessary.
3) To check that you have learned the skills required, practice them again and again until you consistently perform well. There are plenty of questions in this book and in the Textbook, and your teacher may be able to provide you with extra questions.

Problem Solving — Applying Skills to New Situations

These kinds of questions give you more practice at using learned skills, but they also require problem-solving abilities. They are often real-life applications of theoretical problems, for example, **"A distribution company sent out five times more crates of oranges on Tuesday than it did on Monday. Over the two days, it sent out 120 crates of oranges. How many crates of oranges did it send out on Tuesday?"** You need two distinct abilities here: the skills required to solve the equations, and the ability to translate the real-life problem into math. If you are struggling with this type of question, it is very important to pinpoint the cause of the difficulty.

1) If you have difficulty even starting these questions, then you need practice at problem solving, and translating real-world problems into math. Try to get your teacher (or other students) to work through some real-world examples with you. Ask them to start with simple examples, and move on only when you have understood each one.
2) If you can translate the problem into math, but then you solve it incorrectly, you may need to review and relearn the necessary skills. Have a look at the advice on Skills Practice Questions.
3) You may be able to solve real-world problems "in your head," without the need to write down your method. While this is an equally valid way of solving the problem, you should realize that it is important to explain all your steps — if only for the purposes of checking any mistakes later on. If you find this difficult, you could start by explaining your reasoning to someone, and ask them to help you to write that "in math."

Not all Applying Skills Questions will be real-life applications. Some will be Challenge Questions (see below), where you will be asked to apply your skills to different kinds of theoretical problems.

Interesting/Challenging Questions

These are questions designed to interest or challenge you, particularly once you have already mastered the basics. They are usually Applying Skills Questions, but are generally more difficult and often involve several different skills in one question, for example, **"Convert 20 miles per hour into meters per second."**

This is a conversion between units, but here compound units of "meters per second" and "miles per hour" are used, whereas you may have only so far have converted between simpler units, such as "meters" to "miles," or "seconds" to "hours." The skills are the same, but you need to figure out how to apply those skills to a more difficult problem.

These questions are designed to stretch and challenge you, and are the kind that will usually only be set in class, where you can get help from your teacher or other students to work through the problem. In these questions it is not always the math that is more difficult — the questions sometimes involve different ways of thinking. So, even if you struggle with some areas of math, you may still be able to make good headway with some of the Challenge Questions. The most important thing is to not get fazed by them — try applying what you know and see what happens.

Proof Questions

Some questions ask you to use logical arguments to show that a mathematical statement is true or false, for example, **"Show that the sum of the measures of the internal angles in an n-sided shape is $180(n - 2)°$"**

Many people struggle with this idea, and are more comfortable with questions where you need to "find an answer." However, the processes are the same whether you are finding an answer or proving that a given answer is true. Each step should be justifiable, or it may be incorrect.

It is helpful to have a list of useful properties and formulas next to you when you are trying to prove something. If you find yourself unsure of the next step, look down through the list and see if you can apply any of them. If you find something that you think that you can apply, try it and see what happens. If you struggle with this, try to get your teacher (or other students) to work through plenty of examples with you.

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Name _____ Class _____ Date _____

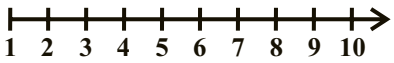
Lesson 1.1.1

Comparing Integers

California Standard: Number Sense 1.1

Example

The natural numbers are used for counting.



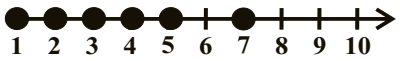
a. Place the numbers {3, 7, 2, 5, 1, 4} on the natural number line.

b. Which of the six numbers is to the right of the other numbers?

c. Which of the six numbers is the greatest?

Solution

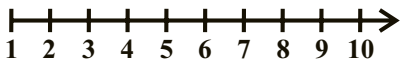
a. Dots are used to show numbers on the number line.



b. **7** is to the right of the other numbers.

c. The numbers are greater as you go further to the right on the number line. 7 is the furthest to the right, so **7** is the greatest of the six numbers.

1. **a.** Place the numbers {2, 5, 1, 9, 7} on the natural number line.



Natural numbers are 1, 2, 3, 4, 5...

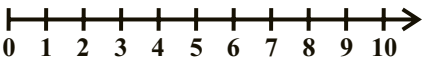
b. Which number is to the right of the other numbers? _____

c. Which number is to the left of the other numbers? _____

d. Which number is the greatest? _____

e. Which number is the least? _____

2. **a.** Place the numbers {5, 0, 3, 6, 1} on the line of whole numbers.



Whole numbers are 0, 1, 2, 3, 4, 5...

b. Which number is to the right of all the numbers? _____

c. Which number is to the left of all the numbers? _____

d. Which number is the greatest? _____

e. Which number is the least? _____

3. In the set of numbers {2, 7, 0, 5, 1), which numbers are:

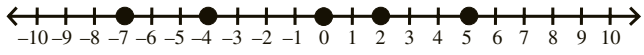
a. natural numbers? _____ **b.** whole numbers? _____ **c.** integers? _____

Example

a. Place the numbers {–4, 5, 0, –7, 2} on the integer number line.
b. Which number is to the right of all the other numbers?
c. Which number is to the left of all the other numbers?
d. Which number is the greatest?
e. Which number is the least?

Solution

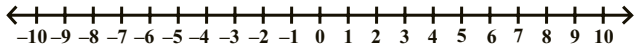
a.



b. 5 is to the right of all the other numbers.
c. –7 is to the left of all the other numbers.

d. The numbers are greater as you go further to the right on the number line, so **5** is the greatest.
e. The numbers are less as you go further to the left on the number line, so **–7** is the least.

4. a. Place the numbers {3, –4, 6, 0, –2} on the integer number line.



b. Which number is to the right of all the other numbers? _____

c. Which number is to the left of all the other numbers? _____

d. Which number is the greatest? _____

e. Which number is the least? _____

f. Which integers are between –2 and –5 on the number line? _____

g. Which integer is between –5 and –7 on the number line? _____

5. Jake said there are no integers between –6 and –7 on the number line.

a. Is Jake's statement correct? _____

b. List 2 integers that have one integer between them on the number line. _____

6. Place the correct inequality sign (<, >) between each pair of numbers.

a. 5 _____ 7 **b.** –4 _____ –2 **c.** –3 _____ 5 **d.** –2 _____ –5

e. 6 _____ 3 **f.** 6 _____ –2 **g.** 0 _____ –3 **h.** 5 _____ 0

i. –4 _____ 0 **j.** 7 _____ 1 **k.** 2 _____ –2 **l.** –3 _____ 3

7. Li said that –4 is less than –2, but Roger said –4 is greater than –2.

a. Who is correct? _____

b. Explain how you know who is correct. _____

_____

Name _____ Class _____ Date _____

Lesson 1.1.2 Adding and Subtracting Integers

California Standard: Number Sense 2.3

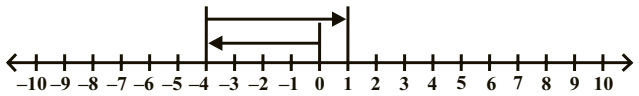
Example

Use the number line to solve –4 + 5.

Solution

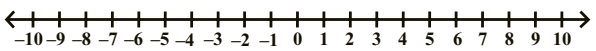
Start by going from 0 to –4, then add 5.
You end up at 1, so –4 + 5 = **1**.

To add a positive integer, move to the right on the number line.

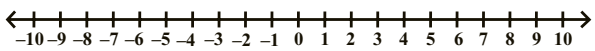


1. Using the integer number line, find the answer to each of the following:

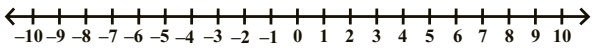
a. –6 + 3 = _____



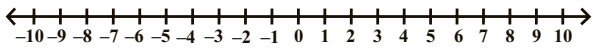
b. –8 + 4 = _____



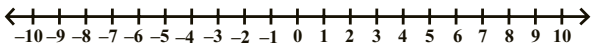
c. –4 + 8 = _____



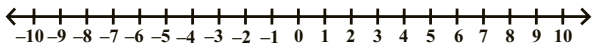
d. 7 + (–3) = _____



e. 3 + (–5) = _____



f. 5 + (–9) = _____



2. Determine the answer to each of the following addition problems:

a. –4 + 7 = _____ **b.** 5 + (–4) = _____ **c.** –3 + 8 = _____ **d.** 7 + (–2) = _____

e. –7 + 11 = _____ **f.** –5 + 6 = _____ **g.** 6 + (–4) = _____ **h.** 9 + (–6) = _____

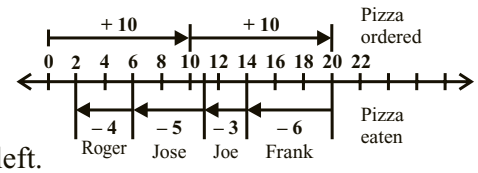
i. 8 + (–6) = _____ **j.** –8 + 11 = _____ **k.** 8 + (–2) = _____ **l.** –5 + 10 = _____

Example

Frank invites three friends over for pizza. He orders 2 large pizzas, each cut into 10 slices.
Frank eats 6 slices, Joe eats 3 slices, Jose eats 5 slices, and Roger eats 4 slices.
How many slices are left?

Solution

You can solve this problem using the number line.
First, add the number of slices Frank ordered.
Then take away the number of slices eaten. There are **2 slices** left.



3. In football, yards gained can be written as positive numbers, and yards lost as negative numbers. Pablo carried the football 6 times during one game. His gains and losses were +7, –5, +4, +3, –2, –6. How many yards did Pablo gain or lose overall during the game?

_____

4. A deep sea diver was 25 feet below the surface. He came up 7 feet, then came up another 8 feet. What level did he end up at?

Example

Solve: 5 – (–3)

Solution

Subtracting a negative number is the same as adding a positive number of the same size.
So 5 – (–3) = 5 + 3 = **8**

5. Solve:

a. 4 + (–7) = _____	**b.** 4 – (–7) = _____	**c.** –6 – (–4) = _____	**d.** –6 + (–4) = _____
e. 5 + 7 = _____	**f.** 5 – 7 = _____	**g.** –7 + 4 = _____	**h.** –7 – 4 = _____
i. 8 + (–5) = _____	**j.** 8 – (–5) = _____	**k.** –3 – (–7) = _____	**l.** –3 + (–7) = _____
m. 5 + 3 = _____	**n.** 5 – 3 = _____	**o.** –2 + 7 = _____	**p.** –2 – 7 = _____

6. Lenny, Luisa, and Kia are going diving in a lake. Lenny goes down 15 feet. Luisa dives 25 feet below Lenny. Kia swims to 7 feet above Luisa. How far below the surface of the lake is Kia?

---

7. Mr Rodriquez has $57 in his checking account.
a. He writes a check for $29. What will his new balance be? Will he be overdrawn?

---

b. He writes a second check for $35. What will his balance be now? Is he overdrawn?

---

8. A mountaineer climbed to an elevation of 1000 feet. He came down the mountain at a rate of 10 feet per minute. What was his elevation after 1 hour?

---

9. The highest temperature in the United States on a particular day was 124 °F. The lowest temperature was 33 °F. What was the range of these temperatures?

Remember — the range is the difference between the two temperatures.

---

10. The range between the high and low temperatures outside Malivai's house was 37 °F. The low temperature was 42 °F. What was the high temperature?

---

11. On Sunday morning when Alicia woke up, the temperature was 50 °F. When she had dinner, the temperature had risen by 33 °F. When she went to bed, the temperature had fallen 21 °F from the previous reading. When she woke up on Monday morning, the temperature had fallen 18 °F from the previous reading. What was the temperature when Alicia woke up on Monday?

---

12. In the Sierra Mountains, the morning temperature was –13 °F. The weather forecaster announced that the temperature would rise 37 °F to reach its high for the day. What will the high temperature be?

Name _____ Class _____ Date _____

Lesson 1.2.1

Multiplying with Integers

California Standard: Number Sense 2.3

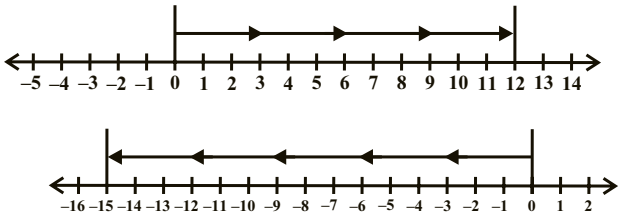
Example

Using the integer number line, solve: **a.** $4 \times 3 = ?$ **b.** $5 \times (-3) = ?$

Solution

a. 12 is 4 times as far away from 0 as 3 is.
So $4 \times 3 =$ **12**

b. –15 is 5 times as far away from 0 as –3 is.
So $5 \times (-3) =$ **–15**

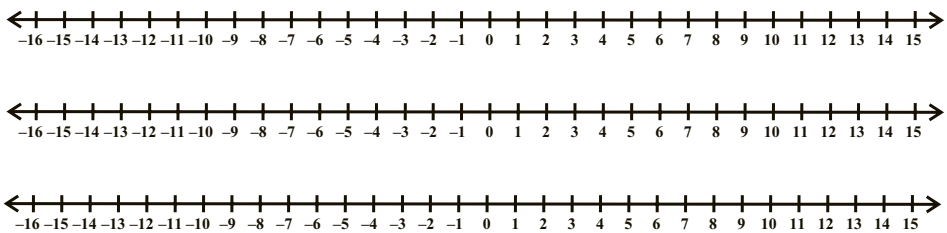


1. Solve each of the following using the integer number line:

a. $4 \times (-4) =$ _____

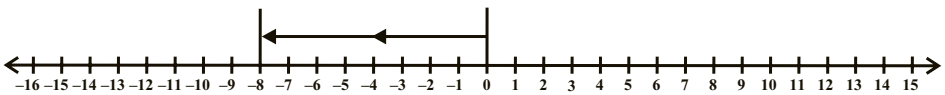
b. $5 \times (-2) =$ _____

c. $3 \times (-5) =$ _____



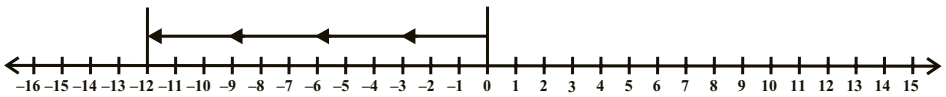
2. For each number line below, write down the multiplication problem shown. Give the factors and the product. The first one has been done for you,

a.



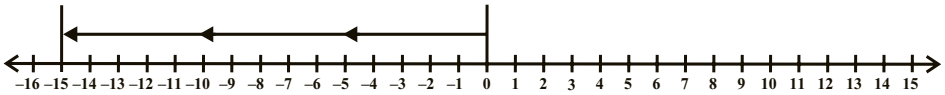
$2 \times (-4) = -8$

b.



_____

c.



_____

3. Solve each of the following multiplication problems:

a. $4 \times (-3) =$ _____	**b.** $-4 \times (-6) =$ _____	**c.** $-9 \times 8 =$ _____	**d.** $6 \times (-9) =$ _____
e. $8 \times 7 =$ _____	**f.** $-5 \times 9 =$ _____	**g.** $7 \times (-9) =$ _____	**h.** $-4 \times (-9) =$ _____
i. $9 \times (-3) =$ _____	**j.** $-6 \times (-8) =$ _____	**k.** $-8 \times 4 =$ _____	**l.** $3 \times 8 =$ _____
m. $-6 \times 7 =$ _____	**n.** $7 \times (-4) =$ _____	**o.** $-3 \times (-7) =$ _____	**p.** $-6 \times 4 =$ _____

4. A deep sea diver can go down 10 feet every minute. If he starts at the surface of the water, what depth can he reach after 7 minutes?

---

5. Jo the mountain climber is at 3700 feet above sea level on a mountain.
She comes down 200 feet every 5 minutes. How far above sea level is she after 1 hour?

---

6. Say whether the following answers will be positive or negative, without working out the solutions:

a. $4 \times (-6) \times (-9) \times 5 \times (-4) =$ ---------- **b.** $27 \times (-6) \times 38 \times 87 \times (-9) =$ ----------

c. $(-65) \times (-43) \times 49 \times (-8) \times 37 =$ ---------- **d.** $(-53) \times 87 \times (-63) \times 96 \times 42 =$ ----------

The number of negatives will determine whether the product is positive or negative.

7. Death Valley, California, is usually very hot. A scientist recorded the maximum temperature in February as 70 °F. Over the next few months, she expects the maximum temperature to rise by 10 °F each month. If this is correct, what will the maximum temperature be in June?

---

8. In January Mr. Reynolds had overdrawn his bank account. The balance was –$20.
For each of the next 4 months Mr. Reynolds overdrew his account by a further $15.
What was his balance at the end of this period?

---

9. The local grocery store has loss leaders on sale every weekend. Loss leaders are items on sale where the store actually loses money on that item, but hopes to attract customers who will buy other items and make up the loss. For its loss leader this weekend, the store is selling chocolate chips at $1.00 per package, but the store actually pays $2.00 per package.

a. How much does the store lose on each package?

---

b. If the store sold 200 packages of chocolate chips, what would their total loss be?

---

10. At the end of the snow season, Mt Whitney has 240 inches of snow. A weather forecaster estimates that, during the next 6 weeks, the snow will melt at a rate of 15 inches per week. How much snow does the weather forecaster predict will be on Mt Whitney after the 6 weeks?

Name _____ Class _____ Date _____

Lesson 1.2.2

Dividing with Integers

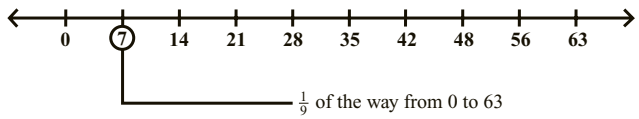
California Standard: Number Sense 2.3

Example

Using the integer number line, solve: $63 \div 9 = ?$

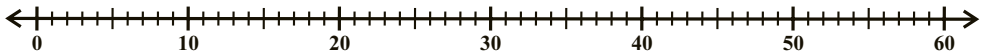
Solution

7 is one-ninth of the way from 0 to 63 on the number line.
So $63 \div 9 = \mathbf{7}$

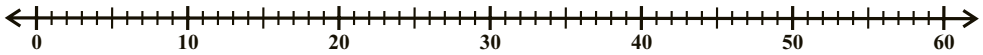


1. Using the number line, solve:

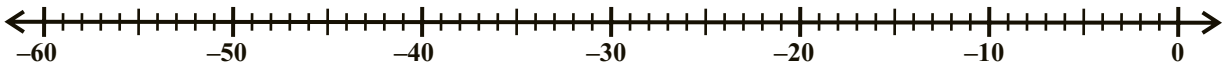
a. $56 \div 7 =$ _____



b. $27 \div 9 =$ _____

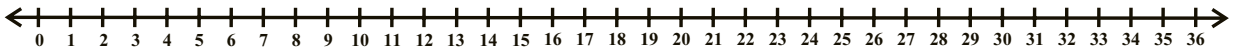


2. Arelia found the value of $-54 \div (-9)$ by making equal groups on a number line.
a. Show on the number line how Arelia found equal groups.



b. What is $-54 \div (-9)$? _____

3. Use the number line below to help you answer this question.



Write a division to find the following numbers. The first one has been done for you.

a. The number that is halfway from 0 to 36 on the number line. $36 \div 2 = 18$

b. The number that is one-third of the way from 0 to 36 on the number line. _____

c. The number that is one-fourth of the way from 0 to 36 on the number line. _____

d. The number that is one-sixth of the way from 0 to 36 on the number line. _____

4. Solve:

a. $64 \div (-8) =$ _____	**b.** $42 \div (-7) =$ _____	**c.** $-36 \div 6 =$ _____	**d.** $-27 \div (-9) =$ _____
e. $28 \div (-7) =$ _____	**f.** $-18 \div (-6) =$ _____	**g.** $-72 \div 8 =$ _____	**h.** $56 \div 8 =$ _____
i. $45 \div 9 =$ _____	**j.** $24 \div (-8) =$ _____	**k.** $-21 \div (-3) =$ _____	**l.** $-40 \div 8 =$ _____
m. $63 \div (-9) =$ _____	**n.** $-24 \div 4 =$ _____	**o.** $-32 \div (-4) =$ _____	**p.** $54 \div 6 =$ _____

5. A clock showed the correct time at 8:00 a.m. However, by 2:00 p.m. the clock showed 1:48 p.m. On average, how much time did the clock lose each hour between 8:00 a.m. and 2:00 p.m.?

---

6. Solve:

a. $81 \div (-9) =$ _____	**b.** $-36 \div 9 =$ _____	**c.** $35 \div 7 =$ _____	**d.** $-48 \div (-6) =$ _____
e. $63 \div (-7) =$ _____	**f.** $-24 \div 6 =$ _____	**g.** $-32 \div (-8) =$ _____	**h.** $54 \div 9 =$ _____
i. $-64 \div 8 =$ _____	**j.** $42 \div (-6) =$ _____	**k.** $36 \div 6 =$ _____	**l.** $-27 \div (-3) =$ _____
m. $28 \div (-4) =$ _____	**n.** $-18 \div (-3) =$ _____	**o.** $-72 \div 9 =$ _____	**p.** $56 \div 7 =$ _____
q. $45 \div 5 =$ _____	**r.** $24 \div (-3) =$ _____	**s.** $-21 \div (-7) =$ _____	**t.** $-40 \div 5 =$ _____

7. Louis is walking down the stairs of a skyscraper. He descends 924 steps in 11 minutes. On average, how many steps does he descend in one minute?

---

8. In the middle of the afternoon, the temperature was 87 °F. Five hours later, the temperature was 62 °F. On average, how much did the temperature change per hour?

---

9. During the month of April, Mrs. Palmer sold $25,000 worth of merchandise at her store. The merchandise cost her $19,000, and the monthly expenses came to $9000. What was her average profit per day?

---

10. High tide was measured at a level of 8 feet. Low tide, 7 hours later, was measured at a level of 1 foot. On average, how much did the tide change every hour?

Name _____ Class _____ Date _____

Lesson 1.2.3

Integers in Real Life

California Standard: Number Sense 2.3

Example

Kiana, Larry, Oscar, and Elizabeth have collected seashells for a school project. Kiana collected 36 shells. Say which operation is needed to solve the following problems, and find the answers.

a. Larry collected 11 fewer shells than Kiana. How many shells did Larry collect?
b. Oscar collected one-third the number of shells that Kiana did. How many shells did Oscar collect?
c. Elizabeth collected twice as many shells as Oscar. How many shells did Elizabeth collect?
d. How many shells did Kiana, Larry, Oscar, and Elizabeth collect in total?

Solution

a. The number you need is 11 fewer than 36, so use **subtraction**: $36 - 11 = 25$.
Larry collected **25 seashells**.
b. To find one-third of 36, use **division**. One-third means divide by 3: $36 \div 3 = 12$.
Oscar collected **12 seashells**.
c. To find twice the number Oscar collected, use **multiplication**: $12 \times 2 = 24$.
Elizabeth collected **24 seashells**.
d. To find the total, use **addition**. $36 + 25 + 12 + 24 = 97$.
In total, they collected **97 seashells**.

1. The angles a flower opened up to at different times of the day are shown in the chart. Write +, –, ×, or ÷ to show the operation needed to solve each question.

a. What was the increase in angle measurement between 6 a.m. and 10 a.m.? _____

b. The angle measurement increased by 20° from 6:00 a.m. to 8:00 a.m. What was the angle measure at 8:00 a.m.? _____

c. The angle increased by 45° between 10 a.m. and 2 a.m. What was the average increase in the angle each hour? _____

d. The angle measurement at 3:00 p.m. was double what it was at 10:00 a.m. What was the measurement at 3:00 p.m.? _____

Time	Angle
6:00 a.m.	20°
10:00 a.m.	65°
2:00 p.m.	110°
6:00 p.m.	65°

2. Mrs. Smith buys 24 pencils. She wants to share them equally between her 4 children, so she uses division to work out how many pencils each child gets.

a. Which number will be the divisor? _____

b. Which number will be the dividend? _____

c. Which number is the quotient? _____

d. How many pencils will each child receive? _____

3. A diver is 120 feet below the surface. He is 6 times as deep as his son, who is just learning to dive. How deep is his son?

---

4. Mr. Jones bought 300 shares of stock for $20.00 per share. He sold the stock 3 months later for $18.00 per share.
a. How much did Mr. Jones lose on each share?

---

b. How much did Mr. Jones lose in total when he sold the stock?

---

5. Death Valley's elevation goes from –282 feet below sea level to over 11,000 feet above sea level. Jamila started at –200 feet, climbed 440 feet, and then came down 110 feet. What was her final elevation?

---

6. Dave does odd jobs in the neighborhood. He did 3 jobs where he earned $5 each, and 4 jobs where he earned $8 each. On his way home, Dave bought 4 cans of soup for $3 each.
How much does he now have left from his jobs?

---

7. Mr. Flint had $364 in his bank account on May 1. During the month he deposited $36 and $89, and he wrote checks for $45, $67, and $123. What was his balance at the end of May?

---

8. Mr. Spencer's checking account has a balance of $135. For each of the next 4 months he withdraws $40. The following month he deposits $100, then for each of the next 3 months he withdraws $30. What is his balance at the end of the 8 months?

---

9. A hawk is flying at 500 feet above the ground. Her prey is flying at 200 feet above the ground. The hawk dives at a rate of 20 feet per second. The prey is rising at 10 feet per second.
a. How long before the hawk catches her prey?

---

b. How far above the ground does the hawk catch the prey?

Name _____ Class _____ Date _____

Lesson 1.3.1

Decimals

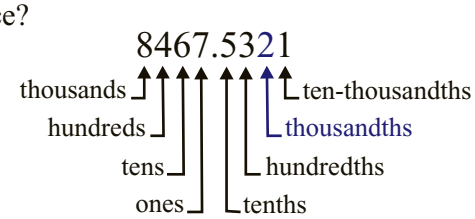
California Standard: Number Sense 1.1

Example

In the number 8467.5321, what digit is in the thousandths place?

Solution

The digit in the thousandths place is **2**.



1. **a.** In the number 45.789, what is the digit in the hundredths place? _____

b. In the number 365.749, what is the digit in the hundreds place? _____

c. In the number 4316.9763, what is the digit in the ten-thousandths place? _____

d. In the number 5697.435, what is the digit in the thousands place? _____

Example

Write 234.589 in expanded form.

Solution

The 200 will come first, then the 30, then the 4 completing the left side of the decimal.
.5 will be next, then .08 to put 8 in the hundredths place, then .009 to put 9 in the thousandths place.
So 234.589 = **200 + 30 + 4 + .5 + .08 + .009**

2. Write 3000 + 400 + 6 + .5 + .08 + .0002 as a decimal. _____

3. A school's budget was $625,093.49. Write the digit that is in the named place.

a. hundreds _____ **b.** hundredths _____

c. ten thousands _____ **d.** ones _____

e. tenths _____

4. The temperature on Mt Lassen on four consecutive days was:

Monday	–9.3 °C
Tuesday	–9.2 °C
Wednesday	–9.7 °C
Thursday	–9.5 °C

Which day was the coldest?

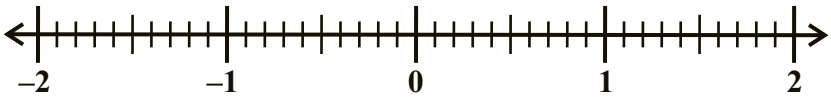
a. Monday ❐ **b.** Tuesday ❐ **c.** Wednesday ❐ **d.** Thursday ❐

5. Which of the following numbers is the furthest from zero?

a. 7.34 ❐ **b.** 7.3 ❐ **c.** –7.43 ❐ **d.** –7.4 ❐

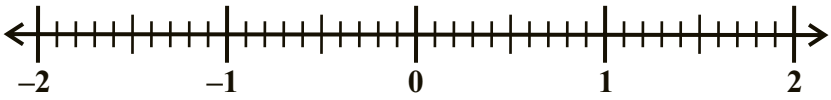
6. The table on the right shows the change in the percentage of students at Timmy's school who rode the bus, for each of the last 5 years.
Plot the numbers from the table on the number line below.

Year	% Change
1	+ 1.3
2	+ 0.8
3	– 0.4
4	– 0.2
5	– 0.6



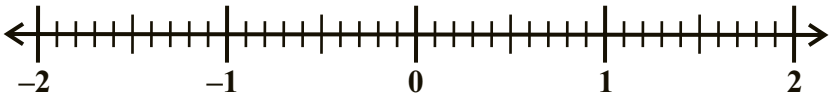
7. Michelle is trying to improve her 400 meter race time.
The change in her time after each of 4 races is shown in the table.
Plot the numbers from the table on the number line below.

Race	Change
1	– 0.13
2	– 0.81
3	– 1.3
4	+ 0.1



8. A farmer recorded the change in the average yield of one of his crops in the table. Plot the numbers from the table on the number line below.

Year	Change in yield
2000	– 1.63
2002	+ 0.38
2004	+ 0.91
2006	+ 0.1



9. Which of these pairs of numbers are both a distance of 4.7 away from 3.2 on the number line?

a. 1.5 and 7.9 ❐ **b.** –1.5 and 7.9 ❐ **c.** –1.5 and –7.9 ❐ **d.** 1.5 and –7.9 ❐

Name _____ Class _____ Date _____

Lesson 1.3.2

Ordering Decimals

California Standard: Number Sense 1.1

1. Joe earned $34.56 at his after-school job. Fran earned $34.59 at her after-school job.

a. Who earned the most money? _____

b. How much more money did the person who earned the most receive?

_____

2. On Monday morning the temperature was –7.3 °C. On Tuesday morning it was –7.5 °C, and on Wednesday morning it was –7.1 °C.

a. Which morning was the coldest? _____

b. Which morning was the warmest? _____

3. Place the correct inequality sign (< or >) between each pair of numbers.

a. 4.56 _____ 4.57 **b.** 6.01 _____ 6.001 **c.** 20 + 4 + .3 + .05 _____ 24.33

d. five and forty-two hundredths _____ five and forty-three hundredths

Example

One day on the coast near San Francisco, the first low tide was –1.5 feet and the second low tide was –1.7 feet.

a. Which tide was lower? **b.** How much lower was the lower tide?

Solution

a. –1.7 is to the left of –1.5 on the number line, so –1.7 is less than –1.5.
So the **second tide** at –1.7 feet was lower.

b. –1.5 – (–1.7) = 0.2
So the second tide was **0.2 feet** lower than the first one.

4. Daisy and Imara are rock climbing in a valley where the elevation is below sea level. Daisy is at –134.6 feet below sea level, and Imara is at –121.9 feet below sea level.

a. Who is higher up the rock face?

_____

b. Find the difference between Daisy's elevation and Imara's elevation.

_____

5. Harry and Janice are using remotely-controlled submarines to collect samples from the sea floor. Harry sends his submarine down to a level of –231.4 feet. Janice's submarine goes down to a level of –231.9 feet. Whose submarine dived further below the surface?

---

6. Order each of the following from greatest to least.

a. 4.65, 4.68, 4.6 ---

b. 17.5, 17.56, 17.53 ---

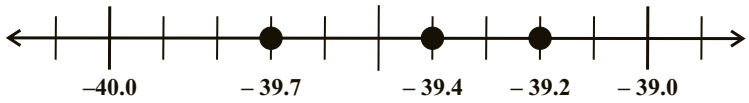
c. 122.6, 121.68, 121.66 ---

Example

Which is the greatest number: –39.2, –39.7, or –39.4?

Solution

You can compare the numbers using the number line.



–39.2 is the furthest to the right on the number line, so **–39.2** is the greatest of the three numbers.

7. Three rock samples were collected by a geologist from a group of caves. He recorded how far below ground he found each sample. Sample X was found at a level of –18.65 meters, and sample Y was found at a level of –18.09 meters. Sample Z was found 0.6 meters below sample Y.

a. At what level did the geologist find sample Z?

---

b. Place the three numbers representing the depths the geologist found the samples at in order from greatest to least.

---

8. Order each of the following from greatest to least:

a. 34.56, 34.52, 34.58 ---

b. 0.2, 0.02, 0.002 ---

c. 4, 4.1, 4.01 ---

d. –58, –57.99, –58.1 ---

e. –6.1, 6.8, –6.4 ---

Name _____ Class _____ Date _____

Lesson 1.4.1

Rounding Numbers

California Standard: Mathematical Reasoning 2.1

Example

Round 23475 to the nearest hundred.

Solution

First underline the place value you are to round to: 23<u>4</u>75.
The digit to the right of the underlined number is 7. 7 is greater than 5, so you need to round up.
23475 rounded to the nearest hundred is **23500**.

1. Use the following set of numbers: 7.842 7.389 8.340 8.5

a. Which of the four numbers is greater than 8 when rounded to the nearest whole number? _____

b. Which of the four numbers is less than 7.8 when rounded to the nearest hundredth? _____

2. Yolanda has $5287.89 in her savings account. What place value did she round to if she rounded the amount to:

a. $5290.00? _____ **b.** $5000.00? _____

c. $5288.00? _____

3. A basketball team scored an average of 98.37 points per game in the first 18 games of the season.

a. What is this number to the nearest ten? _____

b. What is this number to the nearest whole number? _____

c. What is this number to the nearest tenth? _____

d. Which number in a, b, or c is the greatest? _____

4. Round 623,475 to the nearest thousand. _____

5. Round each number to the nearest hundred, then determine whether the original number is greater or less than the rounded number.

a. 5648 _____ **b.** 8234.67 _____ **c.** 27,456.456 _____ **d.** 345,576 _____

_____ _____ _____ _____

6. Round 24.5679 to the nearest hundredth. _____

7. The city of San Francisco, California, had a population of 776,733 in the 2000 census.

a. Round this population to the nearest thousand. ----------

b. How close is the rounded number to the real number? ----------

c. Round the population to the nearest ten thousand. ----------

d. How close is this rounded number to the real number? ----------

8. Round 47,582 to the nearest:

a. ten ---------- **b.** hundred ---------- **c.** thousand ----------

9. Round 5.4638 to the nearest:

a. thousandth ---------- **b.** hundredth ---------- **c.** tenth ----------

10. Mr. Perez went to buy a new car. The price of the car he wanted was $16,489.87.
Mr. Perez said, "I will give you the money rounded to the nearest thousand dollars."
What did Mr. Perez offer?

---

11. Ms. Magnusson goes to buy a boat. The boat she wants is priced at $24,328.89.
She offers the marked price, rounded to the nearest thousand dollars.
What does Ms. Magnusson offer for the boat?

---

12. The city of Redding, California, had a population of 80,865 in the 2000 census. Round the population to the nearest thousand. Is the rounded number larger or smaller than the real population?

---

13. A number that is often used in mathematics is π. $\pi = 3.14159265...$
Round π to the nearest:

a. hundredth ---------- **b.** thousandth ---------- **c.** ten-thousandth ----------

d. For each of these rounded numbers, say whether it is more than or less than the unrounded π.

Name _____ Class _____ Date _____

Lesson 1.4.2

Using Rounded Numbers

California Standards: Mathematical Reasoning 2.1, 2.6

Example

Toby, Farouk, and Vanessa each drew a map of their school.
Toby marked the length of the main school building using the exact figure of 124.82 meters.
Farouk marked the length of the main building as 125 meters.
Vanessa marked the length of the main building as 100 meters.
Which level of rounding is the most suitable?

Solution

People looking at the map probably won't need to know the exact length of the building.
So writing the exact figure is not very useful.
Rounding to the nearest hundred meters would be misleading.
The figure of 100 meters is very different from the real figure, so it isn't a good choice.
125 meters is the most suitable choice. It is close enough to the real figure to give people a good idea of the size of the building, but doesn't give them extra information that they don't need.

1. The time taken for a train journey between two cities was recorded as 1 hour, 51 minutes, 17 seconds.
The rail company is going to use this time to work out its timetable.
Which of the following levels of rounding would be most suitable? Explain your answer.
a. 1 h 51 min 17 s (exact) **b.** 1 h 51 min (nearest minute) **c.** 2 hours (nearest hour)

---

2. The table below lists the mass of a space probe to three different levels of accuracy.
Choose which of the figures would be most suitable in the following situations. Explain your choices.

Mass	Level of accuracy
986.355 kg	Exact figure
986 kg	Nearest 1 kg
1000 kg	Nearest 100 kg

a. A reporter is going to mention the size of the probe in a newspaper headline.

---

b. An engineer at the space agency is making precise calculations to plan the flight path of the probe.

---

c. Jenny is making a poster about the space probe for science club.

Example

Keifer calculated that 9699 ÷ 212 = 79.5.
Use rounding to check whether or not this answer looks reasonable.

Solution

You can check this answer using rounding to the nearest hundred:
9700 ÷ 200 = 48.5
The exact answer to 9699 ÷ 212 should be near to 48.5, so Keifer's answer is **not reasonable**.

3. Tammy's answers to 3 different math problems are shown.
Use estimation to say whether Tammy's answer is reasonable or unreasonable in each case.

a. 1204.504 ÷ 31.4 = 38.36 ----------

b. 9015.4 + 385.6 = 12871.4 ----------

c. 18.053 – 0.6758 = 11.295 ----------

4. Use rounded numbers to estimate the following:

a. The sum of 78.4 and 22.6 ----------

b. The difference between 78.4 and 22.6 ----------

c. The product of 78.4 and 22.6 ----------

d. The quotient, 78.4 divided by 22.6 ----------

5. Lara added the numbers 134 + 23 + 256 + 49 and got 1110. Estimate the sum by rounding to the nearest ten. Is Lara's answer reasonable?

---

6. A bus driver drove the same route for 21 days in a row. The route was 59.4 miles. The driver said she drove 12,000 miles in all. Does the driver's statement look reasonable? Explain your answer.

---

7. The average scores for each round of a 4 round golf tournament are shown in the chart. Jake rounded the scores and got a sum of 279.
Kelly rounded the scores and got a sum of 280.
Explain how both answers could be correct.

Round	Average Score
1	69.43
2	70.15
3	68.32
4	72.41

Name _____ Class _____ Date _____

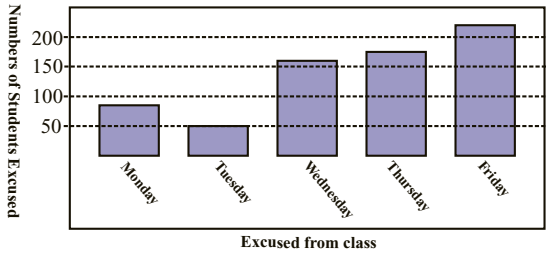
Lesson 1.4.3

Estimation

California Standard: Mathematical Reasoning 2.3

Example

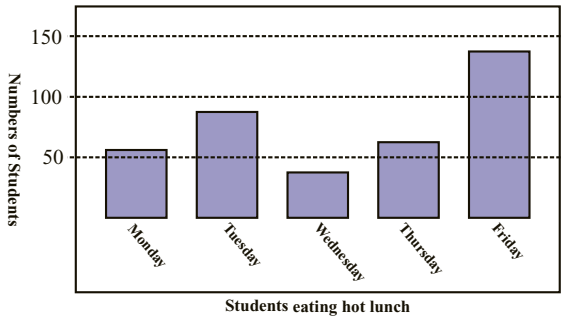
The bar graph shows the number of students excused from classes in one school district during the past week. Estimate the number of students that were excused on Thursday.



Solution

The height of the bar representing Thursday is about halfway between 150 and 200, so a good estimate would be that 175 students were excused on Thursday.

1. The bar graph shows the number of grade 6 students in one school who ate hot lunch each day during the past week.

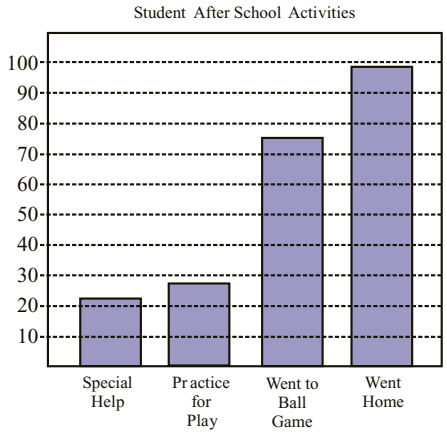


a. Estimate the number of students who ate hot lunch on Tuesday.

_____

b. Estimate the number of students who ate hot lunch on Friday.

_____



2. The graph on the left shows the after school activities of a group of grade 6 students on a particular day.

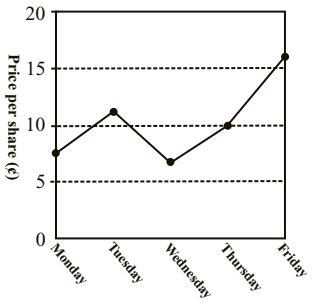
a. From the graph, estimate how many students went to the ball game.

_____

b. Estimate the total number of students who did something other than go home.

_____

3. The line graph shows the price of a stock during the past week.



a. Estimate the price on Monday.

_____

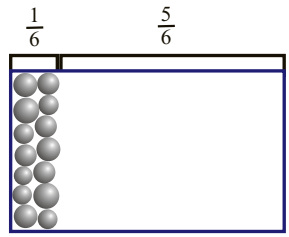
b. Estimate the price on Friday.

_____

c. Estimate how much the price increased over the week.

_____

Example

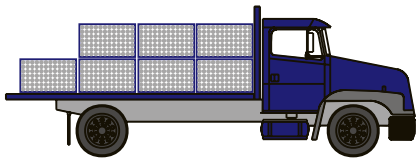


The picture on the right shows a crate which is partially filled with oranges.

a. Estimate the total number of oranges that would fit in the crate.

b. A truck is carrying 100 such crates.
Estimate how many oranges are on the truck.

Solution



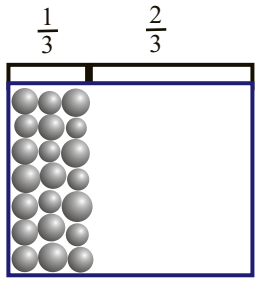
a. The picture shows that 14 oranges fill up about one-sixth of the crate, so the total number of oranges the crate will hold is about $6 \times 14 = 84$ oranges.

b. If each crate holds about 84 oranges, then you can estimate that the truck is holding about $100 \times 84 = 8400$ oranges.

---

4. The picture on the right shows a tray which is partly filled with apples.

a. Estimate the number of apples needed to fill the tray.



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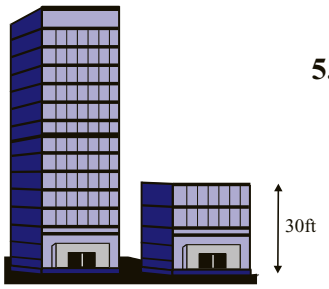
b. A crate holds 3 trays of apples like the one shown.
Estimate the total number of apples that will fit in the crate.

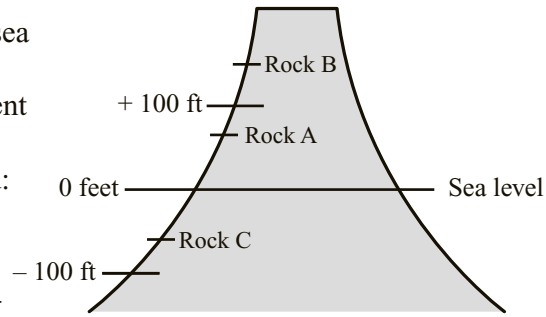


5. Estimate the height of the taller building in the picture on the left.

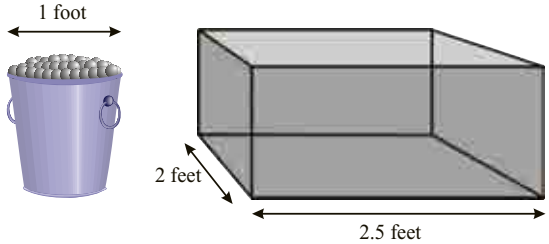
---

6. The volcanic mountain shown on the left is partly below sea level and partly above sea level.
A geologist collected rocks A, B, and C from three different heights on the mountain, as marked on the diagram.
Estimate what height the geologist was at when she found:

a. Rock A ---------- **b.** Rock B ---------- **c.** Rock C ----------



7. The bucket shown below holds 2 pounds of cherries.
Estimate how many pounds of cherries you would need to fill the large box shown.



Name _____ Class _____ Date _____

Lesson 1.4.4

Using Estimation

California Standards: Mathematical Reasoning 2.1, 2.6

Example

Mr. Martz is having a party. He goes to the store to buy snack food.
Is it appropriate for Mr. Martz to estimate the amount of food he needs to buy?

Solution

Mr. Martz doesn't know exactly how much food his party guests will eat. It wouldn't make sense to work out a precise amount in this situation, so it is appropriate for Mr. Martz to use estimation.

1. Mr. Johansen is buying a number of items at the hardware store. He is going to pay by check.
Is it appropriate for him to use estimation to decide how much to pay? Explain your answer.

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2. Cassandra is measuring to replace the glass in one of her windows.
Can the measurements be reasonable estimates or should they be precise? Explain your answer.

---

3. Chico is painting his house. Should he use precise measurements to calculate how much paint he will need? Explain your answer.

Example

Mrs. Smith takes $25.00 to the store. She picks out items costing $3.23, $5.64, $8.98, and $1.23.
Use front-end estimation to check whether or not Mrs. Smith has enough money to pay for her items.

Solution

First, add the front-end digits:
$3.00 + $5.00 + $8.00 + $1.00 = $17.00
Now make a rough estimate of the value of the remaining digits:
$0.98 is about $1.00;
$0.23 + $0.64 + $0.23 is about another $1.00.
The total cost of the items is about $17.00 + $1.00 + $1.00 = $19.00.
So Mrs. Smith does have enough money to pay for the items.

4. Use front-end estimation to estimate the total cost of three books priced at $9.39, $7.75, and $8.88.

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5. Michael wants to buy four shirts, priced at $22.10, $31.97, $26.09, and $24.80.
Use front-end estimation to estimate the total cost of the four shirts.

---

6. Kadeem is working out how long it will take him to walk across town, so he can tell his friend when to meet him. Is an estimate or an exact figure more suitable in this situation? Explain your answer.

---

7. A group of 7 people went to lunch. They decided to split the bill equally. The bill was $140.00. Is estimation necessary here? Explain your answer.

---

8. Nina scored 87, 73, 96, and 82 on her four English tests. Is it appropriate for her teacher to use rounding to add up the four test scores for his records? Explain your answer.

---

9. Daniel wants to mention the number of cars in the city of Los Angeles in a report. Is it more appropriate for him to use an exact number or an estimate? Explain your answer.

---

10. Mrs. Franklin is about to go to the drug store to pick up her prescriptions. She has $25 in her purse, and she knows the prices will be $3.68, $6.74, and $8.57. Use front-end estimation to work out whether or not Mrs. Franklin should stop at the ATM for more money on the way to the drugstore.

---

11. Lisa takes $5.00 to school each day to buy lunch at the cafeteria. She wants to buy the following lunch: sandwich $3.40, milk $0.75, dessert $1.85. Use front-end estimation to work out whether Lisa has enough money.

---

12. Ricardo's mother sent him to the store with $20.00 to do some shopping. Ricardo bought items priced at $4.98, $2.27, $5.14, and $5.62. Use front-end estimation to check whether he has enough money.

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13. Amisha's bank account had a balance of $214.87. She wrote checks for $25.17, $18.24, and $41.62.

a. Use front-end estimation to estimate the total value of the three checks.

---

b. Amisha calculates that her new bank balance will be $99.84. Using your estimate from part a., say whether or not you think her calculation is correct.

Name _____ Class _____ Date _____

Lesson 2.1.1

Variables

California Standard: Algebra and Functions 1.2

Example

DaMarcus has 4 more apples than Ana.
a. What extra information do you need to be able to figure out how many apples DaMarcus has?
b. Choose a letter to represent this unknown number.

A **variable** is assigned to unknown numbers.

Solution

a. To find how many apples DaMarcus has, you need to know the number of apples that Ana has.
b. Let the variable *a* represent the number of apples that Ana has.

1. John is 5 years older than his brother.

a. What extra information do you need to be able to find John's age?

---

b. Choose a letter to represent this unknown number.

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2. Jane is 23 days older than Wanda.

a. If you want to know Jane's date of birth, what extra information will you need?

---

b. Choose a letter to represent this unknown number.

---

3. Felipe earns $8.00 per hour in his job.

a. What extra information will you need to find the total amount Felipe earned yesterday?

---

b. Choose a letter to represent this unknown number.

---

4. Juanita works 30 hours a week. She works the same number of hours each working day, and the same number of days each week.

a. You want to know how many hours per day Juanita works. What extra information do you need?

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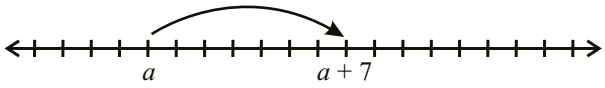
b. Choose a letter to represent this unknown number.

Example

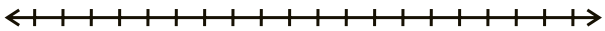
The number 7 is added to the variable a.
Show this on a number line.

Solution

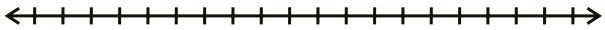
Mark a on a number line, and then move 7 to the right.



5. Represent "3 more than n" on the number line.

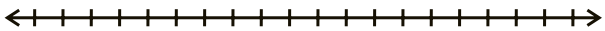


6. Show "$z + 16$" on the number line.



7. Show "$y - 13$" on the number line.

8. Represent "4 times y" on the number line, if the variable y is positive.



You'll need to mark zero on the number line too.

9. A triangle has two sides with the same length, and one side 5 inches longer.

a. Identify the unknown quantity needed to find the length of the longer side.

---

b. Choose a letter to represent the variable. ---

10. A delivery person delivered 8 fewer packages in the morning than in the afternoon.

a. Identify the unknown quantity needed to find the number of packages delivered in the morning.

---

b. Choose a letter to represent the variable. ---

Name _____ Class _____ Date _____

Lesson 2.1.2

Expressions

California Standard: Algebra and Functions 1.2

Example

Rewrite the statement, "a number is increased by 3," as an expression.

Solution

Replace "a number" with the variable n. Variables are usually represented by letters of the alphabet.

Since the statement says, 'increased', the operation needed is addition.

Then you can rewrite the expression as $\boldsymbol{n + 3}$.

In Exercises 1–4, write an expression for each word problem.

1. A number, f, is increased by 25.

---

2. A number, R, is doubled.

---

3. Zachary is weighing two packages. The first package weighs 15 times as much as P, the weight of the second package. How much does the first package weigh?

---

4. The temperature at 6:00 p.m. was 35 °F higher than t, the temperature at 8:00 a.m. What was the temperature at 6:00 p.m.?

---

5. Evaluate each of the following expressions.

a. $x + 5$, for $x = 4$

---

b. $8 - y$, for $y = 3$

---

c. $5r$, for $r = 7$

---

d. $56 \div p$, for $p = 8$

In Exercises 6–9, rewrite each expression without the × symbol.

6. $21 \times b$ ----------

7. $12 \times c$ ----------

8. $-9 \times r$ ----------

9. $-27 \times s$ ----------

In Exercises 10–11, circle the algebraic expression in each set of expressions.

10. $6 + 92$ $15 \div 8$ $y \times 13$

11. $x - 46$ $28 + 48$ $\frac{32}{8}$

12. Carly had a total of d dollars to buy party decorations. She spent one-fifth of her money on streamers. Write an expression for the amount she spent on streamers.

---

13. Jem is laying the table. There are already p plates on the table when Jem adds the last 8 salad plates. Write an expression for the final number of plates on the table.

Name _____ Class _____ Date _____

Lesson 2.1.3

Multi-Variable Expressions

California Standard: Algebra and Functions 1.2

1. Write each of the following expressions using words.

a. zx _____

b. $u \div v$ _____

c. $5r - 3s$ _____

d. $uv - w$ _____

2. How many terms are there in each of the following expressions?

a. $x + y$ _____ **b.** $x + 3$ _____

c. y _____ **d.** $a + b - c$ _____

e. $5a + b - c$ _____ **f.** $a - 2b + c + d$ _____

3. State the number of variables and the number of terms in each of the following expressions.

a. $3xy$ _____ **b.** $4x + 5y$ _____

c. $r + s + t$ _____ **d.** $5x + 3$ _____

4. Evaluate each of the following expressions when $r = 7$, $s = 4$, $t = 12$.

a. $r + s$ _____ **b.** $\frac{t}{s}$ _____

c. rs _____ **d.** $t - r$ _____

5. Evaluate each of the following expressions using $r = 6$, $s = 10$, $t = 12$, $u = 15$.

r^2 means $r \times r$

a. $r + s - u$ _____ **b.** $u - t + r$ _____

c. $r^2 - s - u$ _____ **d.** $rs + u$ _____

6. Given $x = 2$, $y = 4$, $z = -5$, evaluate the expression $\frac{y}{x} - z$. _____

Example

A sports club gains income from donations ($\$d$), memberships ($\$m$) and selling goods ($\$s$).
a. Write an expression for the club's income.
b. Evaluate your expression using: $d = 143.85$, $m = 455$ and $s = 162.50$

Solution
a. Add the three variables together to get the total income: $\boldsymbol{d + m + s}$
b. Replace the variables with their values: $143.85 + 455 + 162.50 =$ **$761.35**

7. The elevator in the Empire State Building, New York, starts at floor 30, goes up r floors and then comes down s floors.

a. Write an expression to show which floor the elevator ends up at. _____

b. Evaluate your expression using $r = 37$ and $s = 24$. _____

8. Mr. Smith went to the store with a $20 bill and spent x dollars on meat and y dollars on vegetables.

a. Write an expression to show how much change Mr. Smith was given. _____

b. Evaluate your expression using $x = \$4.50$ and $y = \$7.25$. _____

9. In June, Angela worked a hours at b dollars per hour. She then had c dollars deducted for income tax.

a. Write an expression to show how much Angela was paid, after tax. _____

b. Evaluate your expression using $a = 20$, $b = 8.00$, and $c = 14.55$. _____

10. During the summer, Alex spent u hours a week for v weeks in the gym. He also ran for a total of w hours over the summer.

a. Write an expression for how long Alex spent exercising over the summer. _____

b. Evaluate your expression using $u = 5$, $v = 12$ and $w = 40$. _____

11. Francis spends r hours running, w hours lifting weights, and k hours kicking a soccer ball every day.

a. Write an expression to show how long Francis spends training every day. _____

b. Evaluate your expression using $r = 0.75$, $w = 0.5$, and $k = 0.6$. _____

12. Jim was on a weight loss program. His starting weight was s pounds, and he lost p pounds a week for w weeks to reach his ideal weight of i pounds.

a. Write an equation to show Jim's ideal weight, i. _____

b. Evaluate your expression using $s = 175$, $p = 2$ and $w = 14$. _____

Name _____ Class _____ Date _____

Lesson 2.1.4

Order of Operations

California Standards: Algebra and Functions 1.3, 1.4

Example

Mr. Sanchez gave his class the problem $4 + 3 \times 5$. Simone got the answer 35, but Joey got 19. Who is right? Explain your answer.

Solution

Simone did the addition first: $4 + 3 \times 5 = 7 \times 5 = 35$.
Joey did the multiplication first: $4 + 3 \times 5 = 4 + 15 = 19$.
Joey is right, because multiplication should be done before addition.

There is a set of rules to make sure everyone gets the same answer for the same problem.
First, do all the work within parentheses.
Second, evaluate any exponents.
Third, multiply and divide in order from left to right.
Fourth, add and subtract in order from left to right.

1. Wendell needed to evaluate the expression $(5^2 - 3 \times 8) \times 2$.

a. Circle the operation below that must be done first.

Addition Subtraction Multiplication

Division Exponents

b. Wendell said that he first needed to find the product of 8 and 2. Is Wendell's statement correct? Explain your answer.

---

2. Using the rules for the order of operations evaluate each of the following expressions.

a. $5 + 6 - 7$ _____ **b.** $3 + 4 \times 5 - 6$ _____

c. $5 + 24 \div 6 - 8$ _____ **d.** $30 \div 5 \times 6$ _____

3. Jose works for $9.00 an hour and gets paid every three weeks. The first week he worked 7 hours, the second week he worked 9 hours, and the third week he worked 10 hours.

a. Write an expression to show how much Jose's paycheck will be. _____

b. Evaluate your expression. _____

4. Evette sells magazine subscriptions. Today, she has sold 18 subscriptions for $12 each and n subscriptions for $15 each. Write an expression to show how much money Evette has made today.

---

5. Denny read a new novel. When he started the book, he read 15 pages a day for 8 days. After that, he read 10 pages a day for a further 7 days until he had finished the novel. Write and evaluate an expression to find the number of pages in the novel.

---

6. Evaluate each of the following expressions when $r = 3$, $s = 15$, $t = 5$, $u = 2$.

a. $\frac{s}{r} + tu$ ---

b. $r^{u} - \frac{s}{t} + ru$ ---

c. $u^{r} - (t - u) + su$ ---

7. Using the rules for the order of operations, evaluate each of the following expressions.

a. $3 + (24 \div 6)^2 - 8$ ---

b. $10 - (4 - 3)^2 + 5$ ---

c. $(15 \div 5)^2 + (3 \times 2)^2 - (2 \times 15)$ ---

8. Mr. Smith goes to the grocery store with a $20.00 bill. He buys seven apples at $0.25 each, five pints of milk at $0.75 each, and two heads of lettuce at $1.25 each.

a. Write an expression for the amount of change that Mr. Smith will receive.

---

b. Evaluate your expression.

---

9. Evaluate each of the following expressions.

a. $12 - (5 - 3)^3 + 5$ ---

b. $(7 - 4)^3 - 2(3)^2 + (4 + 2)^2$ ---

c. $6 + 3^2 - 2^3 + 4^2$ ---

Name _____ Class _____ Date _____

Lesson 2.2.1

Equations

California Standard: Algebra and Functions 1.1

Example

Antonio is on a 100-mile bicycle trip. He has completed 57 miles so far, and has another x miles to go. Write an equation to represent this.

Solution

When Antonio's ride is complete, he will have cycled $(57 + x)$ miles.
This is equal to the total length of the trip, which is 100 miles.
You can write this as the equation $\mathbf{57 + x = 100}$.

1. Write equations for each of the following statements.

a. A number added to 23 is equal to 57. _____

b. The sum of a number and 36 equals 94. _____

c. When a number is added to 47 the sum is equal to 66. _____

d. 87 is equal to the sum of a number and 43. _____

2. Use "guess and check" to find the value of y in each of the equations below.

a. $y + 4 = 22$ _____

b. $972 \div y = 108$ _____

3. Use "guess and check" to find the value of t in each of the equations below.

a. $s \times r \times t = 50$ and $s = 2$, $r = 5$ _____

b. $(t + 6) + y = 40$ and $y = 10$ _____

4. Alyssa is using "guess and check" to find the value of z in the equation $15 \times z = 500$. Alyssa says that z must be between 10 and 20. Is Alyssa's statement correct? Explain your answer.

_____

5. Jaden spent h dollars at the dry cleaners and k dollars at the supermarket. He spent m dollars altogether. Write an equation to show how much Jaden spent.

---

6. 192 students visited last year's pep rally. This year m more students came than last year. Alfonso wrote the equation $t = (192 + m) + 192$ for the total number of students who came during the two years. Is Alfonso's equation right? Explain your answer.

---

7. Hannah is making bread. The first batch she made weighed 17 ounces. The second batch she made weighed w ounces. The two batches weighed 43 ounces in total. Write an equation to describe the situation.

---

8. The height of a building under construction was h feet on Monday. On Tuesday the height increased by 24 feet to reach the finished height of t feet. Write an equation linking h and t.

---

9. Malika paid a total of t dollars for gas when she filled her tank. She bought 16 gallons of gas at a cost of n dollars per gallon. Write an equation linking t and n.

---

10. William and Julian are playing a video game. William scored 1500 more points than Julian. If William scored w points, write an equation about t, the total number of points scored by William and Julian.

Name _____ Class _____ Date _____

Lesson 2.2.2

Manipulating Equations

California Standard: Algebra and Functions 1.1

Keep the equations balanced — do the same to both sides.

1. Find the following expressions using the information provided.

a. If $z = 14$, find $z + 8$. _____

b. If $p = 9$, find $72 \div p$. _____

c. If $l = 21$, find $2 \times l$. _____

d. If $m - n = 7$, find $m - n + 2$. _____

e. If $k + t = 13$, find $k + 9 + t$. _____

f. If $20v = 20$, find $20v \div 20$. _____

2. Fill in the missing parts of the following.

a. $y + z = 12$

$y + z - 6 =$ _____

b. $5 + y = -z$

$3 \times (5 + y) =$ _____

In Exercises 3–5, explain what step of the work is incorrect.

3. $x + 8 = 11$

$2 \times (x + 8) = 11 + 2$ _____

4. $15x = 9$

$15x \div 3 = 9$ _____

5. $x - 7 = y$

$x - 7 - 3 = y + 3$ _____

6. Fill in the missing parts of the following:

a. $s - 7 = t$

$s =$ _____

b. $s + t = 17$

$t =$ _____

7. Tamika and Juan each tried manipulating the equation $x + 27 = 73$. Their work is shown below.

Tamika	**Juan**
$x + 27 = 73$	$x + 27 = 73$
$x + 27 - 15 = 73 - 15$	$x + 27 + 73 = 73 + 27$

Have either of them made a mistake? If so, what have they done wrong?

_____

8. Isabel and Janie each tried manipulating the equation $x - 17 = y$. Their work is shown below.

Isabel	**Janie**
$x - 17 = y$	$x - 17 = y$
$2x - 17 = 2y$	$x - 17 + 12 = y + 12$

Have either of them made a mistake? If so, what have they done wrong?

Example

Sylvia buys 3 tennis balls for \$$b$ each, and 1 tennis racket for \$$r$.
She spends \$28.87 in total, so she writes: $3b + r = 28.87$

Sylvia's friend Francisco decides to buy twice as many tennis balls and rackets as Sylvia.

Manipulate Sylvia's equation to represent Francisco's purchase.

Solution

Francisco bought twice the number of tennis balls and rackets that Sylvia did, so multiply the left-hand side of the equation by 2.
This means that you also need to multiply the right-hand side by 2, so you get:

$2 \times (3b + r) = 2 \times 28.87$ or $2 \times (3b + r) = 57.74$

9. At the pet store, large goldfish cost \$$x$, and small goldfish cost \$$y$. Pedro bought 2 large and 3 small goldfish for \$18, so he wrote an algebraic equation to show this: $2x + 3y = 18$.
Kerry bought 3 times as many of each type of fish as Pedro did.
Manipulate Pedro's equation to show this.

---

10. Jessica covered her wall using 2 strips of wallpaper of length p feet, and 5 strips of length q feet. She used 37 feet of wallpaper, so she wrote: $2p + 5q = 37$.
The next day, she covered another wall. This was larger than the previous one, requiring 8 feet more wallpaper. Manipulate the equation to show this.

---

11. Ivan bought 7 hats for \$$h$ each, and 4 scarves for \$$s$ each.
He spent a total of \$94.25, so he wrote: $7h + 4s = 94.25$.
Ivan returned 1 hat, and got a refund of \$$h$. Manipulate the equation to show this.

---

12. Large bags of flour weigh m grams, and small bags of flour weigh n grams.
If $m + 4n = 2550$, find the total weight of 2 large and 8 small bags of flour.

Name _____ Class _____ Date _____

Lesson 2.2.3

Solving + and – Equations

California Standard: Algebra and Functions 1.1

1. Solve each of the following equations.

a. $x + 22 = 44$ _____ **b.** $55 + y = 92$ _____

c. $z + 5.8 = 11.4$ _____ **d.** $6.9 + w = 13.7$ _____

e. $u + 4.3 = -6.2$ _____ **f.** $x + 87 = 45$ _____

g. $-4.7 + y = -8.4$ _____ **h.** $56 + z = -45$ _____

2. Solve each of the following equations.

a. $m - 15 = 9$ _____ **b.** $n - 47 = 99$ _____

c. $p - 7.3 = 9.4$ _____ **d.** $q - 6.9 = 2.5$ _____

e. $r - 4.7 = 2.3$ _____ **f.** $-19.4 = s - 13.8$ _____

g. $-13.5 = t - 10.7$ _____ **h.** $u - (-6.4) = -5.8$ _____

Example

Will withdrew $45.00 from his savings account leaving $189.00 remaining in the account.
a. Write an equation in the form $x - a = b$ that can be used to find the amount in Will's account before the withdrawal.
b. Solve the equation.

Solution

a. $x - 45 = 189$

b. $x - 45 + 45 = 189 + 45$

$x = 234$ **There was $234.00 in Will's account before the withdrawal.**

3. Write an equation in the form $x - a = b$ for each of the following statements.

a. The difference between a number and 35 is 18. _____

b. When 25 is subtracted from a number, the difference is 21. _____

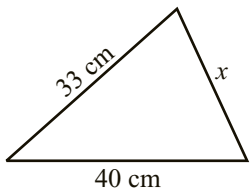
c. You have a number and you subtract 36 to give 47. _____

d. 43 equals a number less 26. _____

4. The perimeter of the triangle shown is 90 cm.

a. Write an equation in the form $x + a = b$ that can be used to determine the length of side x. _____

b. Calculate the length of side x. _____



5. The girls basketball team needs $150.00 to buy 3 new basketballs. They have raised $93.00 so far.

a. Write an equation in the form $x + a = b$ that can be used to find how much more they need to raise.

---

b. Solve the equation.

---

6. Charles writes a check for $23.00, which will leave $138.00 in his account.

a. Write an equation in the form $x - a = b$, where $\$x$ is the amount in his account before the check is paid.

---

b. Solve the equation.

---

7. Mr. Li needs $750 to buy new aluminum bats for his little league team. So far, they have raised $475.

a. Write an equation in the form $x + a = b$ that can be used to find how much more they have to raise.

---

b. Solve the equation.

---

8. A restaurant used 25 kg of flour on Friday. At the end of Friday, the restaurant had 57 kg of flour in stock. Write and solve an equation of the form $x - a = b$ to find how much flour the restaurant had in stock at the start of Friday.

---

9. Mrs. Lopez goes to the store with a $20 bill. She spends $16.20 on groceries.
Write and solve an equation of the form $x + a = b$ to calculate the amount of change she receives.

---

10. Amanda has $130.47 in her checking account after writing a check for $34.25.
Write and solve an equation of the form $x - a = b$, where x is the amount in Amanda's account before the check was written.

---

11. You are given the expression $3a - 4$. If the expression is equal to 14, what is the value of a?

Name _____ Class _____ Date _____

Lesson 2.2.4

Solving × and ÷ Equations

California Standard: Algebra and Functions 1.1

Example

A running track around a football field is 0.25 of a mile long.

a. Write an equation in the form $ax = b$, where x is the number of laps a runner must make to run 5 miles.

b. Solve the equation to find the number of laps in 5 miles.

Solution

a. $0.25x = 5$

b. $0.25x \div 0.25 = 5 \div 0.25$

$x = 20$ **A runner must make 20 laps to run 5 miles.**

1. Penelope and Clive each solved the equation $4x = 124$ as shown.

Penelope	Clive
$4x = 124$	$4x = 124$
$4x \div 4 = 124 \div 4$	$4x - 4 = 124 - 4$
$x = 31$	$x = 120$

Who solved the problem correctly? Explain your answer

---

2. Solve each of the following equations.

a. $3s = 15$ _____ **b.** $5t = 75$ _____

c. $9u = 108$ _____ **d.** $15v = 255$ _____

e. $2.4w = 16.8$ _____ **f.** $-6x = 48$ _____

g. $-4.6y = -27.14$ _____ **h.** $5.7z = -22.8$ _____

3. Write an equation in the form $ax = b$ to describe each of the following statements.

a. The product of 7 and a number equals 42. _____

b. When 9 is multiplied by a number, the product is 72. _____

c. 8 and a number are multiplied giving the answer 56. _____

d. 48 is the product of 6 times a number. _____

4. Each of the following is the solution to an equation with the form $ax = b$.
For each solution, write an equation in the form $ax = b$ that would give that solution.

a. $m = 7$ _____ **b.** $n = -2.4$ _____

c. $p = 3.9$ _____ **d.** $q = -4.6$ _____

5. Darnell and Melinda each solved the equation $\frac{x}{5} = 9$.

Darnell	**Melinda**
$\frac{x}{5} = 9$	$\frac{x}{5} = 9$
$\frac{x}{5} \times 5 = 9 \times 5$	$\frac{x}{5} \div 5 = 9 \div 5$
$x = 45$	$x = 1.8$

Who solved the problem correctly? Explain your answer

---

6. Solve each of the following equations.

a. $\frac{r}{7} = 9$ ---

b. $\frac{s}{6} = 8$ ---

c. $\frac{t}{-9} = 6$ ---

d. $\frac{u}{8} = -7$ ---

e. $\frac{v}{-7} = -7$ ---

f. $\frac{w}{2.3} = 9$ ---

g. $\frac{x}{8} = 3.4$ ---

h. $\frac{y}{4.3} = 5.7$ ---

7. Write an equation of the form $\frac{x}{a} = b$ for each of the following statements.

a. A number divided by 5 equals 8. ---

b. The quotient of a number and 7 equals 9. ---

c. When a number is divided by –4 it equals –9. ---

d. 4 equals the quotient of a number and –7. ---

8. Each of the following is the solution to an equation with the form $\frac{x}{a} = b$.
For each solution, write an equation in the form $\frac{x}{a} = b$ that would solve to give that solution.

a. $m = 72$ ---

b. $n = 56$ ---

c. $p = -32$ ---

d. $q = 42$ ---

9. Harvey is at the store to buy canned vegetables. The cans cost \$0.75 each. Write and solve an equation of the form $ax = b$ to find how many cans Harvey can buy if he has \$10.00.

---

10. Mr. James is buying sweets for his son's birthday party. There will be 12 children at the party, and each child will be given 7 sweets in a party bag. Write and solve an equation of the form $\frac{x}{a} = b$ to find how many sweets Mr. James needs to buy.

---

11. Mrs. Hilton goes to the store with \$20.00 to buy bread. The loaves she chooses cost \$1.79. Write and solve an equation of the form $ax = b$ to show how many loaves of bread Mrs. Hilton can buy.

Name _____ Class _____ Date _____

Lesson 2.2.5

Graphing Equations

California Standard: Algebra and Functions 1.1

In Exercises 1–4, explain how to solve each equation using the inverse operation.

1. $y - 52 = 7$ _____

2. $15y = 60$ _____

3. $y \div 5 = 90$ _____

4. $y + 4 = 17$ _____

Solve each of the equations in Exercises 5–8.

5. $-12 + z = 20$ _____

6. $5f = 40$ _____

7. $q - (-9) = -15$ _____

8. $p \div 8 = -8$ _____

Example

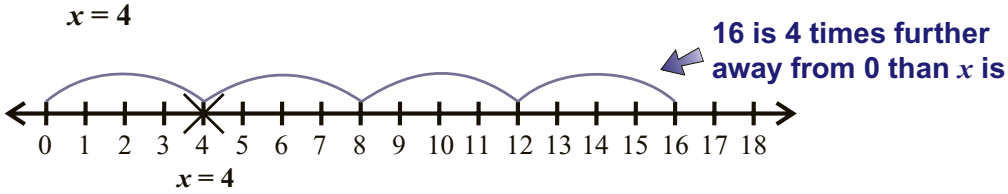
Solve the equation $4x = 16$. Confirm the result using a number line.

Solution

$4x = 16$

$4x \div 4 = 16 \div 4$

$x = 4$



Solve each of the equations in questions 9–10 and graph the results on the number lines given.

9. $x + 3 = 13$

10. $9x = 18$

11. The length of Marco's box car, l, was 15 centimeters less than the length of Jake's box car. Jake's box car was 380 centimeters long. Write an equation to show the relationship between l and the length of Jake's box car, then solve it using the number line.

12. Tara worked 13 hours this week. Next week she is scheduled to work t more hours than this week. She will work a total of 34 hours over the two weeks. Write an equation to show the relationship between t and the hours Tara works, then solve it using the number line.

13. Jen has 280 stamps in her collection. She wants to put 14 stamps on each page of her album. How many pages will she use? Write an equation to answer this, then solve it using a number line.

Name _____ Class _____ Date _____

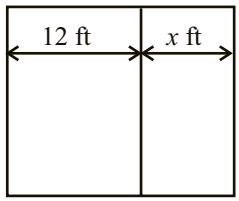
Lesson 2.3.1

Expressions About Length

California Standards: Algebra and Functions 1.2, 1.3, 3.1, 3.2

Example

New carpets are needed for the two rooms in the diagram.
What is the total width of the two rooms?

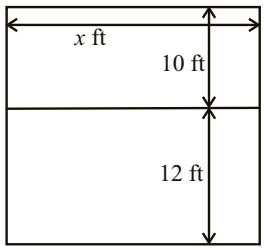


Solution

The two widths added together will give the total width.
Total width = **$(x + 12)$ feet**

1. Raul wants to put a banner around the perimeter of each of the two rooms in the diagram.

Perimeter is the distance around any polygon.



a. What is the perimeter of each room?

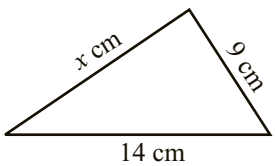
---

b. How long must the banner be in total?

---

2. Write an expression for the perimeter of the triangle in each diagram.

a.



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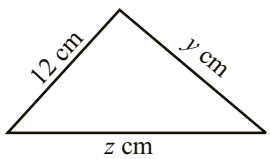
b.



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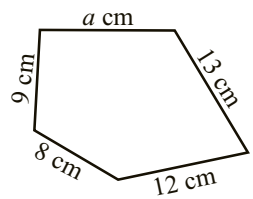
3. Write an expression for the perimeter of the polygon in each diagram.

a.



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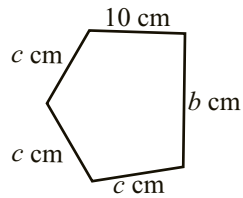
b.



---

4. The length of one side of a square is x cm. What is the perimeter of the square?

---

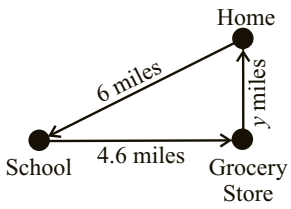
5. Determine the perimeter of a rectangle with width 6 feet and length 11 feet.

The perimeter of a rectangle = $2w + 2l$.

6. Determine the perimeter of a rectangle with length 13 cm and width w cm.

---

7. Mrs. Borne drops her daughter at school, goes to the grocery store, and then returns home. The total round trip, as shown in the diagram, is 16.8 miles. Calculate the distance, y, between the grocery store and Mrs. Borne's house.



8. Determine the perimeter of an equilateral triangle where the length of one side is 20 feet.

---

9. A scalene triangle has a perimeter of 65 cm. Two of the sides measure 23 cm and 27 cm. What is the length of the third side?

---

10. The perimeter of a quadrilateral is 100 inches. The lengths of its sides are 30 in., 30 in., 20 in., and x in. Calculate the value of x.

---

11. The width of a rectangle is 4 inches less than its length. The perimeter of the rectangle is 32 inches. What are the width and length of the rectangle?

---

12. The perimeter of a rectangle is 36 ft. The length is 4 ft more than the width. Calculate the length and width of the rectangle.

Name _____ Class _____ Date _____

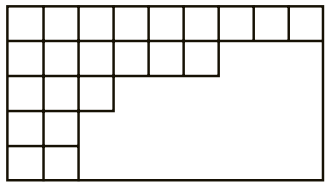
Lesson 2.3.2

Expressions About Areas

California Standards: Algebra and Functions 1.2, 1.3, 3.1, 3.2

Example

Mr Vavalidis is tiling the wall above his bath. The diagram shows the wall and the tiles Mr Vavalidis has put up so far. Write and solve an expression for the number of tiles that Mr Vavalidis will need altogether.



Solution

The wall measures 9 tiles by 5 tiles.

$5 \times 9 = 45$

Mr Vavalidis will need 45 tiles.

1. Write an expression for the area of each of the following rectangles.

a. Base 7 cm and height 9 cm _____

b. Base 4 in. and height x in. _____

c. Base y ft and height 7 ft _____

d. Base p yd and height q yd _____

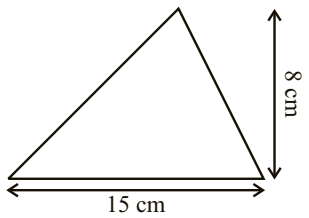
2. Evaluate the area of a rectangle with base b in. and height h in. using the values given.

a. $b = 4$ and $h = 9$ _____ **b.** $h = 19$ and $b = 7$ _____

c. $b = 5$ and $h = 22$ _____ **d.** $b = 14$ and $h = 14$ _____

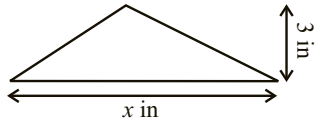
3. Write and evaluate an expression for the area of each of the following triangles.

a.



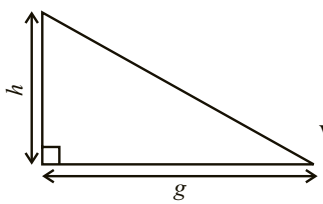
_____

b.



when $x = 12$ _____

c.



when $g = 7$ yd and $h = 4$ yd _____

4. The area of a rectangle is 120 ft^2, and its length is 12 ft. Write and solve an equation to find its width.

---

5. The area of a triangle is 16 in^2, and its base is 8 in. Write and solve an equation to find its height.

---

6. A rectangle has a length of 9 in. and a width of h in.

a. Write an expression for the area of the rectangle. ---

b. Solve your equation when the area of the rectangle is 72 in^2. ---

7. A square has side lengths of l in.

a. Write an expression for the area of the square. ---

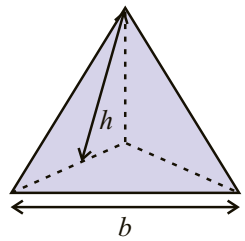
b. Solve your equation when the area of the square is 49 in^2. ---

8. The triangular pyramid in the diagram has four triangular faces, each with height h and base b.

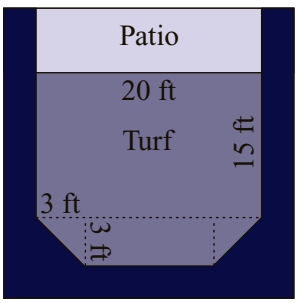
a. Write an expression for the area of one face of the pyramid.

---

b. The total surface area of the pyramid is 48 in^2 and the base of each face is 4 in. Solve your expression to find the height of each triangular face.

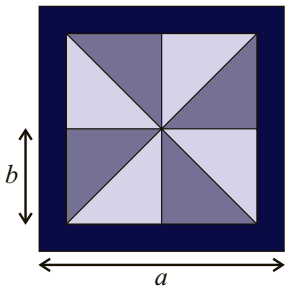


9. Mrs. Casey is having turf laid in her garden. She is trying to calculate the area of turf she will need using the diagram shown. Mrs. Casey has divided the area of turf into simple shapes to make the problem easier. Write and solve an expression for the total area of turf Mrs. Casey needs.



Flowerbed

10. Julianne is making a patchwork quilt. The diagram shows a plan for one of the squares she will make. The pattern consists of 4 medium and 4 light blue isosceles triangles on a dark blue background. Write expressions for the total area of each color in the pattern.



a. Light blue ---

b. Medium blue ---

c. Dark blue ---

Name _____ Class _____ Date _____

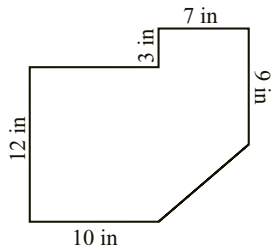
Lesson 2.3.3

Finding Complex Areas

California Standards: Algebra and Functions 3.1, Mathematical Reasoning 1.3

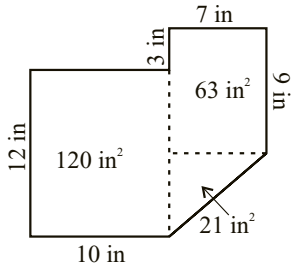
Example

Find the area of the complex shape in the diagram.



Solution

Divide the complex shape into simple shapes:



Calculate the area of each simple shape separately:

$10 \times 12 = 120$

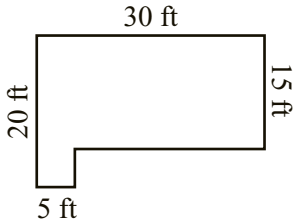
$9 \times 7 = 63$

$(12 + 3 - 9) \times 7 \div 2 = 6 \times 7 \div 2 = 21$

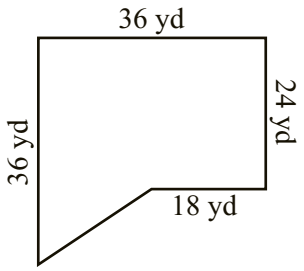
Sum to find the total area: $120 + 63 + 21 =$ **204 in²**

In Exercises 1-4, find the area of the complex shape in each diagram.

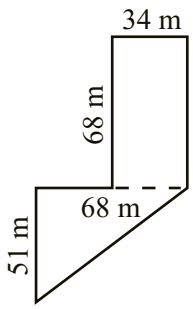
1.



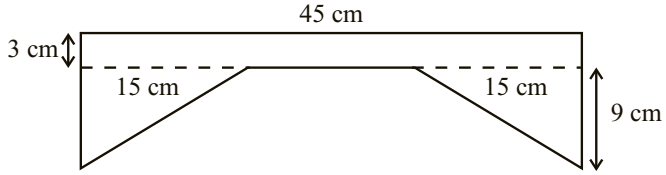
2.



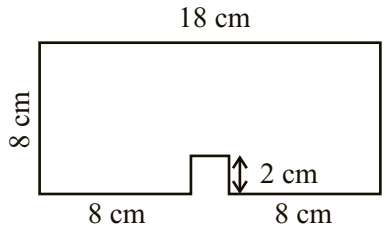
3.



4.



5. Find the area of the shape in the diagram in two different ways. Show your work.



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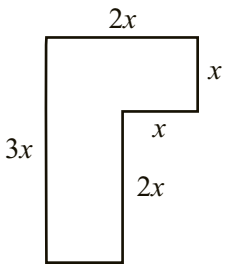
6. Which method in Exercise 5 was easier? Explain your answer.

---

7. Write an expression for the area of the shape in the diagram.



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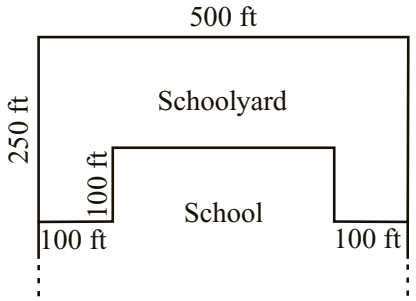
8. The shape of the schoolyard next to a school is shown in the diagram. Find the area of the schoolyard.



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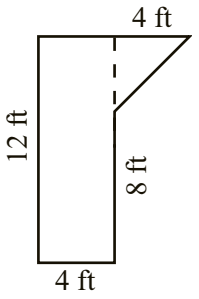
9. The Anderson family have had part of their garden paved over. The dimensions of the paved area are shown in the diagram. Find the area of the paved part of the garden.



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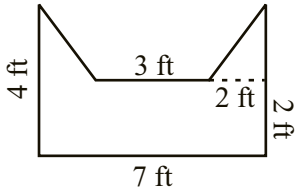
10. The diagram shows a plan view of a design for a desk. Calculate the area of the top of the desk.



Name _____ Class _____ Date _____

Lesson 2.3.4

The Distributive Property

California Standard: Algebra and Functions 1.3

1. Using the distributive property of multiplication over addition, evaluate $5(6 + 4)$. Show your work.

---

2. Evaluate the following expressions using the distributive property of multiplication over addition.

a. $5(10 + 6)$ ---

b. $7(20 + 11)$ ---

c. $4(15 + 13)$ ---

3. Evaluate the following expressions using the distributive property of multiplication over subtraction.

a. $8(20 - 7)$ ---

b. $9(30 - 8)$ ---

c. $12(40 - 25)$ ---

4. Evaluate each of the following expressions using the distributive property. Show your work.

a. $5(-6 + 8)$ ---

b. $-4(8 - 4)$ ---

c. $6(7 + (-7))$ ---

d. $-8(-4 - (-6))$ ---

5. Find the value of x in each of the following equations using the distributive property.

a. $4(x + 3) = 4 \times 5 + 4 \times 3$ ---

b. $x(5 - 7) = 6 \times 5 - 6 \times 7$ ---

c. $8(4 + x) = 8 \times 4 + 8 \times 3$ ---

d. $3(x - 7) = 3 \times 4 - 3 \times 7$ ---

6. Evaluate the following expressions using the distributive property. Show your work.

a. 7×42 ---

b. 9×61 ---

7. Use the distributive property to determine the value of r in each of the following equations.

a. $r(4 + 7) = 9 \times 4 + 9 \times 7$

b. $5(-3 + 8) = 5r + 5 \times 8$

c. $7(6 - r) = 7 \times 6 - 7(-8)$

d. $r(-3 - (-9)) = (-3)(-4) - (-9)(-4)$

Example

Kelly's cell phone calls cost \$0.25 per minute at the weekend. She made a 10 minute phone call on Saturday and a 23 minute phone call on Sunday. Write an expression for the total cost of the calls, then rearrange and solve using the distributive property. Show your work.

Solution

Total cost = $0.25 \times 10 + 0.25 \times 23$ ← **Call on Saturday cost 0.25×10** / **Call on Sunday cost 0.25×23**

Total cost = $0.25(10 + 23)$ ← **Rearrange using distributive property**

Total cost = 0.25×33

Total cost = 8.25

Kelly's weekend calls cost a total of \$8.25.

8. Mr. Roberts takes his family to the movies. He buys 2 child tickets at \$3.00 each and 2 adult tickets at \$6.00 each. Write an expression for the total cost of the tickets, then rearrange and solve using the distributive property. Show your work.

9. Mr. Sexton is buying lunch for himself and his 3 children. For each person, he buys a sandwich at \$3.45, salad at \$2.30, and a drink at \$1.85. Write an expression for the total cost of the lunch, then rearrange and solve using the distributive property. Show your work.

10. Mrs. Morris and her 4 children are visiting a museum. Entry to the museum costs \$4 per person. The children all choose a souvenir costing \$2.50 each. Write an expression for the total amount the family spent at the museum, then rearrange and solve using the distributive property. Show your work.

11. A car rental company charges a flat rate of \$25.00 plus \$10.00 per hour for the first 5 hours and then \$7.50 for every additional hour. Write an expression for the cost of renting a car for 10 hours, then rearrange and solve using the distributive property. Show your work.

Name _____ Class _____ Date _____

Lesson 2.3.5

More Distributive Property

California Standard: Algebra and Functions 1.3

1. The distributive property can be expressed using the equation $a(b + c) = ab + ac$.
Verify the distributive property using the following sets of values. Show your work.

a. $a = 10$, $b = 2$, and $c = 3$ _____

b. $a = 5$, $b = 6$, and $c = 9$ _____

c. $a = 12$, $b = 5$, and $c = 7$ _____

2. Use the distributive property to evaluate each of the following expressions.

a. $5(10 - 2)$ _____

b. $-3(4 + 8)$ _____

c. $9(4 - (-6))$ _____

d. $-8(-3 + 2)$ _____

3. Pauline mentally evaluated 17×5 and got 75 as the answer.
Use the distributive property to check whether or not Pauline is right.

_____

4. Rewrite the following expressions using the distributive property.

a. $p \times (3 + q)$ _____ **b.** $9 \times (n - 4)$ _____

Example

Jeremy rented an apartment costing $385 per month for 6 months. Show how the total rental cost of the apartment could be evaluated mentally, using the distributive property.

Solution

Total cost = 385×6
Total cost = $(300 \times 6) + (80 \times 6) + (5 \times 6)$ **Rearrange using distributive property**
Total cost = $1800 + 480 + 30$ **Evaluate each product separately**
Total cost = 2310
The apartment cost $2310 to rent for 6 months.

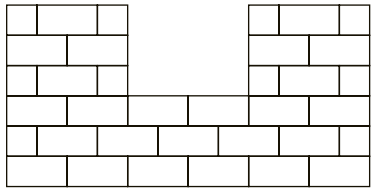
5. Use the distributive property to show how the following expressions could be evaluated using mental math.

a. 7×14 ---------- **b.** 22×3 ----------

6. Amy buys 5 ball point pens costing \$0.99 each. Write an expression of the form $a(b - c)$ to show how the total cost of the pens could be evaluated using mental math.

---

7. The diagram shows part of a brick wall that Allen is building. The finished wall will contain 18 bricks in each of the bottom 3 rows and 10 bricks in each of the top three rows.



a. Write an expression in the form $a(b + c)$ to show how many bricks Allen will use in total.

---

b. Calculate the number of bricks Allen will use.

---

8. Jessica bought 2 packs of yogurts containing 6 pots and 2 packs with only 4 pots of yogurt.

a. Write an expression of the form $a(b + c)$ to show how many pots of yogurt Jessica bought.

---

b. Evaluate your expression.

---

9. In one basketball game, Lyndell scored 3 two-point shots and 4 three-point shots.

a. Write an expression of the form $a(b + c)$ to show how many points Lyndell scored in total.

---

b. Calculate the total number of points Lyndell scored.

Name _____ Class _____ Date _____

Lesson 2.3.6

Squares and Cubes

California Standards: Algebra and Functions 1.2, 3.1, 3.2

Example

Use exponents to rewrite the following expressions in a shorter form.

a. 7×7

b. $4 \times 4 \times 4$

Solution

Exponents show how many times to multiply a number by itself.

a. $7 \times 7 = \mathbf{7^2}$

b. $4 \times 4 \times 4 = \mathbf{4^3}$

1. Write and evaluate an expression that is equivalent to 6^3.

2. The area of a square is $s \times s = s^2$, where s is the length of one side of the square. Calculate the areas of squares with sides of the following lengths.

a. 7 cm

b. 12 in.

c. 6 yd

d. 4 ft

3. What is the surface area of a cube with one edge measuring 4 cm?

4. Calculate the volume of a cube that measures 4 cm along one edge.

5. Calculate the surface area and volume of cubes whose edges measure each of the following lengths.

a. 5 in.

b. 3 ft

c. 10 cm

d. 7 yd

6. Mr. Everett is decorating a room in his house that measures 9 ft by 9 ft by 9 ft.

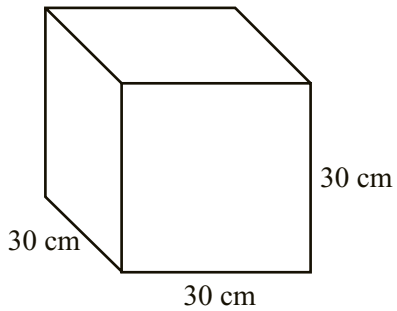
a. How many square feet of wall does Mr. Everett have to paint? ----------

b. What is the volume of Mr. Everett's room? ----------

7. The area of a square is 144 ft^2. What is the perimeter of this square?

---

8. Mrs. Ortega used cement blocks to build a pedestal in the shape of a cube for a statue, as shown in the diagram. What is the volume of the pedestal? Show all your work.



---

9. The volume of a cube-shaped box is 512 in^3. What is the length of one of the edges of this box?

---

10. The surface area of a cube is 216 in^2. What is the length of one of the edges of this cube?

Name _____ Class _____ Date _____

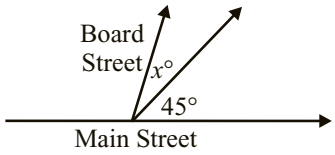
Lesson 2.3.7

Expressions and Angles

California Standards: Algebra and Functions 1.2, 3.1, 3.2

Example

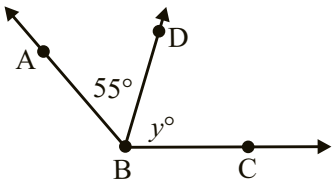
Two streets intersect with Main Street as shown. Write an expression for the measure of the angle at which Board Street intersects Main Street.



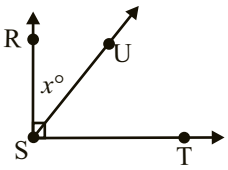
Solution

$(x + 45)°$

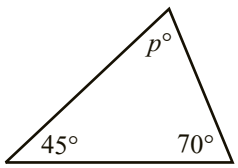
1. Write an expression for the measure of angle ABC.



2. Write an expression for the measure of angle UST.



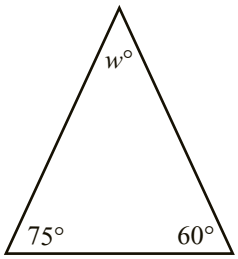
3. **a.** Use the diagram to the right to write an equation involving p.



b. Solve your equation.

The sum of the measures of the angles in a triangle is 180°.

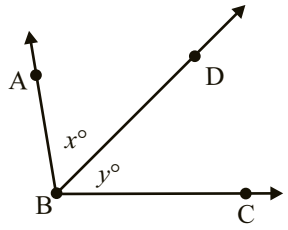
4. Write and solve an equation for w in the triangle to the right.



5. A triangle has angles measuring $u°$, $v°$, and $w°$. Write an equation for w involving u and v.

---

6. If the measure of angle ABC = 100°, and $x = 57$, what is the value of y?



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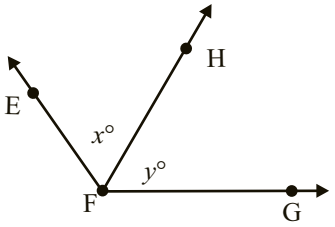
7. If the measure of angle EFG = $r°$, write an equation for x.



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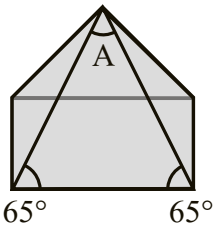
8. The angles of a pyramid depend on its height. The taller the pyramid, the larger the base angles. In the pyramid pictured on the right, what is the measure of angle A, at the top of the pyramid?



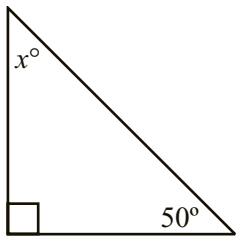
---

9. A triangle has angles measuring 62°, $x°$ and $y°$. Write an equation for the value of x.

---

10. Orrin is making a bookend in wood shop. The diagram shows the side view of the bookend.

a. Write an equation involving x.



---

b. Calculate x.

Name _____ Class _____ Date _____

Lesson 2.4.1

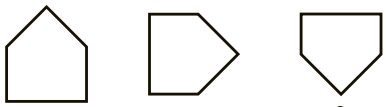
Analyzing Problems

California Standards: Mathematical Reasoning 1.1, 2.4

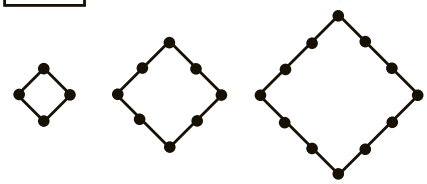
1. Draw the next figure in each series of geometric drawings.

a.

b.



c.



2. Give the next three numbers in each of the following number sequences and explain the pattern.

a. 1, 3, 7, 15, 31, 63, ... _____

b. 1, 1, 2, 3, 5, 8, 13, 21, ... _____

c. 1, –2, 4, –8, 16, –32, ... _____

d. 1, 4, 9, 16, 25, ... _____

e. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$, ... _____

3. How many dots will there be in Figure 10 if the following pattern is continued?

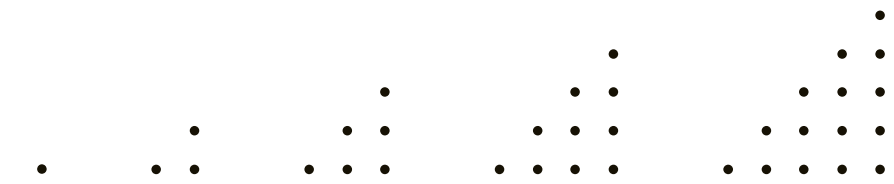


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

_____

Example

Hector was 45 years old 16 years ago. Using an equation, determine Hector's current age.

Solution

Let x = Hector's current age.
Since 45 is 16 less than Hector's current age, $x - 16 = 45$.
Now add 16 to both sides of this equation: $x - 16 + 16 = 45 + 16$.
Simplify this to get: $x = 61$.
Hector's current age is 61 years old.

4. At the morning stocktake, a restaurant had 189 gallons of milk in the refrigerator. 25 gallons of milk were added to the refrigerator during the day. At the end of the day, 204 gallons of milk remained in the refrigerator. How many gallons of milk were used during the day?

---

5. Mr. Gunsberg bought 3 basketballs costing $29.99 each and 2 soccer balls costing $24.99 each. He handed the clerk seven $20 bills. How much change should Mr. Gunsberg have received?

---

6. Richie goes to the auto parts store to buy 4 new tires. He has three $50.00 bills to spend. The tires Richie wants cost $39.95 each.

a. How many of these tires can Richie buy?

---

b. How much change will he receive?

---

7. Karen saves $0.25 out of every dollar she earns. So far, she has saved $40.00. Describe the relationship in this problem using an equation, then solve the equation.

---

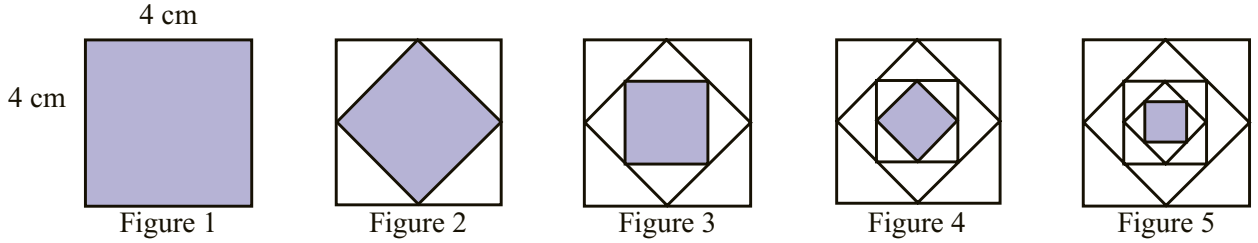
8. The old high school building was 45 years old 15 years ago. Write and solve an equation that expresses the relationship between the school's current age and its age 15 years ago.

---

9. What are the next three numbers in the following number sequence?

1, 5, 17, 53, 161, 485, ---------- ---------- ----------

10. If the pattern below was continued, what would be the area of the colored square in Figure 9?



Name _____ Class _____ Date _____

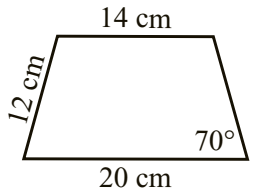
Lesson 2.4.2

Important Information

California Standards: Mathematical Reasoning 1.1

1. Identify any pieces of information given that aren't needed to answer the following questions.

a. Find the perimeter of the isosceles trapezoid in the diagram.



---

b. Joe drives 45 miles up Interstate 5 to take his friend to dinner. Each dinner costs $9.90. How much does Joe spend on dinner?

---

c. A pack of frozen corn costs $1.89 and a pack of frozen peas costs $2.09. If Mr. Walker buys two packs of frozen corn, how much change does he get from a $5.00 bill?

---

2. Identify the important piece of information not given that is needed to answer the following questions.

a. Mrs. Jones buys two large shirts costing $12.95 each and three small shirts. How much change will she get from a $50 bill?

---

b. If Mr. Scott buys a car costing $17,687.00 plus tax, how much does he spend altogether?

---

c. It is 14 miles from Jim's house to Kate's house. If Jim leaves his house and drives at an average speed of 20 mi/h, what time will he arrive at Kate's house?

Example

The following set of clues describe a single number.
Say whether there is too much or too little information to determine the number.

I am between 40 and 70.
I am a multiple of 9.
My first digit is greater than my second digit.

Solution

List all multiples of 9 between 40 and 70: 45, 54, 63
Delete those with a smaller first digit than second digit: 54, 63

After using all the clues given, two possible numbers are left — there is not enough information to determine the single number described.

Too little information.

3. The set of clues in each of the following questions leads to a single number.
For each set of clues, say whether there is too much or too little information to determine the number.

a. I am a multiple of 9.
I am between 50 and 100.
My second digit is less than 2.

---

b. If you add 14 to me, you get a number between 30 and 50.
I am a multiple of 13.
The sum of my two digits is 8.

---

c. If you multiply me by 2, you get a number between 10 and 20.
I am a multiple of 3.
I have only one digit.

---

4. A magic square is a 3×3 grid containing the numbers 1–9.
Every row, column, and diagonal has to sum to 15.
Do you have enough information to complete this magic square?

8		6
	5	

---

5. Do you have enough information to complete this magic square?

	9	
3	5	

---

6. Say whether too much or too little information is given to answer the following questions.
If there is too much information, say which information you don't need.
If there is too little information, say what extra information you need.

a. Gaby gave birth to a baby boy weighing 7 lb 6 oz on May 5th 1996.
If Gaby was 27 years old when she gave birth, in what year will her son be half her age?

---

b. Mr. and Mrs. Hausmann live in Los Angeles. They have two sons — Eric, who lives in Boston, and Dirk, who lives in Atlanta. Plane tickets from Los Angeles to Atlanta cost $320 each.
How much will it cost Mr. and Mrs. Hausmann to visit Eric?

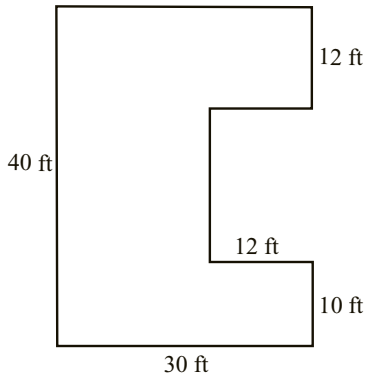
Name _____ Class _____ Date _____

Lesson 2.4.3

Breaking Up a Problem

California Standards: Mathematical Reasoning 1.3, 2.2, 2.4, 2.5, 2.7

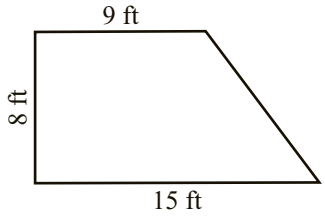
1. Mrs. Fryer wishes to put new carpet throughout the first floor of her house as shown in the diagram. How many square feet of carpet will she need? Show your work.



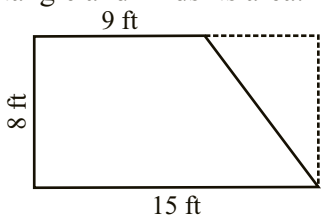
2. Mary babysat for 4 hours on Friday and received $18.00. She babysat for another 14 hours over the weekend, for the same hourly rate as on Friday. How much money did she receive altogether? Show your work.

3. Mrs. O'Leary went to the store to buy clothes for her two children. She found shirts for $5.00 and $3.00. Mrs. O'Leary took $15.00 to the store. She came back with less than $3.00. How many of each type of shirt could she have bought?

4. Eduardo wishes to find the area of the trapezoid shown below.



Eduardo does not recall how to find the area of a trapezoid so he makes a rectangle and finds its area.



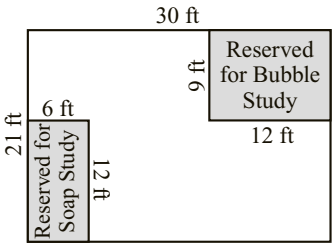
Explain how Eduardo could determine the area of the trapezoid using the rectangle.

5. What is the ones digit in 3^{100}? Explain how you determined your answer.

6. Determine the ones digit in 7^{16}. Explain how you determined your answer.

---

7. The science club meets in Professor Stamford's laboratory every week. Two sections of the room are reserved for Professor Stamford's research, as shown in the diagram. The rest of the room can be used by the science club. Calculate the area of the room that the science club can use.



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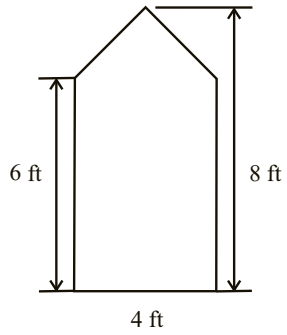
8. Calculate the area of the window shown in the diagram. Include your work to show how you found the area.



Example

Mary-Anne bought a mango costing \$0.65 using nickels, dimes and quarters. Describe the possible combinations of the coins she might have paid with.

Solution

Nickels	1	2	4	6
Dimes	1	3	2	1
Quarters	2	1	1	1

Using a table shows the different combinations clearly

9. Mrs. Smith goes to the department store to buy clothes for her granddaughter Kaitlin. At the department store, shirts cost \$8.00 each and pairs of pants cost \$13.00 each. Mrs. Smith has a \$50.00 bill to spend. Describe the possibilities of what she can buy if she wants to take home less than \$6.00 in change.

---

10. Jane has \$0.85 in her pocket in nickels, dimes and quarters, including at least one of each coin. How many different ways can the \$0.85 be made up? Explain how you determined your answer.

Name _____ Class _____ Date _____

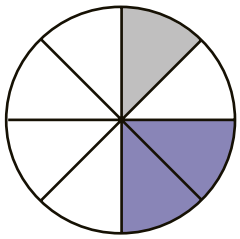
Lesson 3.1.1

Understanding Fractions

California Standard: Number Sense 1.1

Example

The circle below is separated into 8 equal parts. What fraction of the circle is shaded blue, and what fraction is shaded gray?



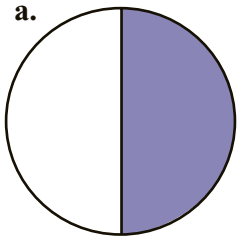
Solution

1 part out of 8 is gray, so $\frac{1}{8}$ **is shaded gray.**

2 parts out of 8 are blue, so $\frac{2}{8}$**, or** $\frac{1}{4}$**, is shaded blue.**

1. The diagrams below represent which fractions?

a.



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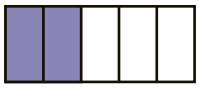
b.



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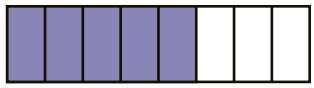
c.



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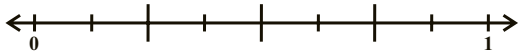
2. Draw figures which represent the following fractions.

a. $\frac{2}{3}$ **b.** $\frac{5}{6}$ **c.** $\frac{4}{9}$

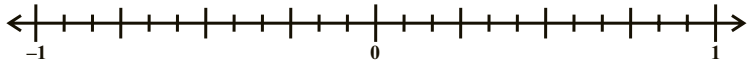
3. Place $\frac{5}{8}$ and $\frac{3}{4}$ on the number line.



Which is greater, $\frac{5}{8}$ or $\frac{3}{4}$? ----------

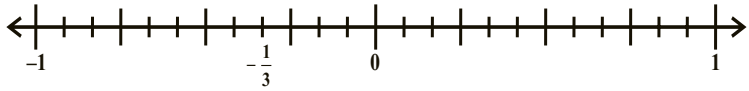
Example

Place $-\frac{1}{3}$ on the number line.

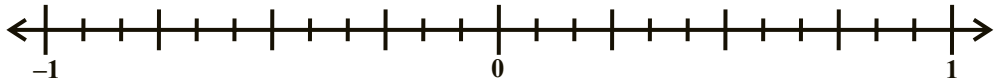


Solution

$-\frac{1}{3}$ is one-third of the distance between 0 and –1.



4. Mark the following numbers on the number line: $\frac{3}{4}$ $-\frac{3}{4}$ $\frac{1}{2}$ $-\frac{1}{2}$ $\frac{2}{3}$ $-\frac{2}{3}$



5. Write one of the symbols, <, =, or >, between the two fractions below to make a true statement.

$\frac{2}{3}$ ---------- $\frac{7}{12}$

6. Place one of the three symbols, <, =, or >, between each pair of fractions.

a. $\frac{2}{3}$ --------- $\frac{3}{4}$ **b.** $\frac{2}{3}$ --------- $\frac{1}{2}$ **c.** $\frac{2}{3}$ --------- $\frac{4}{6}$ **d.** $\frac{1}{3}$ --------- $\frac{1}{2}$

e. $-\frac{1}{2}$ --------- $-\frac{2}{6}$ **f.** $-\frac{3}{4}$ --------- $-\frac{5}{8}$ **g.** $-\frac{3}{4}$ --------- $-\frac{6}{8}$ **h.** $-\frac{1}{3}$ --------- $-\frac{2}{9}$

7. Order the following fractions from least to greatest: $\frac{2}{3}, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}$

----

8. Order each set of fractions from least to greatest.

a. $\frac{3}{4}, \frac{7}{8}, \frac{1}{2}$ ---------- **b.** $\frac{5}{6}, \frac{2}{3}, \frac{1}{2}$ ---------- **c.** $-\frac{5}{5}, -\frac{4}{5}, -\frac{3}{5}$ ---------- **d.** $-\frac{1}{3}, -\frac{2}{9}, -\frac{1}{4}$ ----------

9. Cody runs track. He runs races that are 1 mile long, $\frac{1}{3}$ of a mile long , $\frac{1}{4}$ of a mile long, $\frac{1}{2}$ of a mile long, and $\frac{3}{4}$ of a mile long. Order these 5 distances from the shortest to the longest.

Name _____ Class _____ Date _____

Lesson 3.1.2

Improper Fractions

California Standard: Number Sense 1.1

Example

Mr. Johnson has a pipe 16 ft long. He wishes to cut it into 5 equal lengths. How long is each length?

Solution

Each piece is $16 \div 5$ feet long. $16 \div 5$ can be written as $\frac{16}{5}$, so **each piece is $\frac{16}{5}$ ft long**.

Any fraction where the numerator is larger than or equal to the denominator is called an **improper fraction**.

1. 8 sandwiches are divided among 5 people. How many sandwiches will each person get? Give your answer as a fraction.

---

2. Joe shared \$63.00 between his 5 children. How much money did each child get? Give your answer as a fraction.

Example

What is $\frac{8}{5}$ as a mixed number?

Solution

$\frac{8}{5} = \frac{5}{5} + \frac{3}{5} = 1 + \frac{3}{5} = \mathbf{1\frac{3}{5}}$.

3. Convert $\frac{11}{7}$ to a mixed number. _____

4. Convert $\frac{21}{4}$ to a mixed number using division. _____

5. Convert each of the following to mixed numbers.

a. $\frac{9}{2}$ ----------

b. $\frac{17}{3}$ ----------

c. $\frac{33}{5}$ ----------

6. Convert each of the following to improper fractions.

a. $7\frac{3}{4}$ ----------

b. $4\frac{5}{8}$ ----------

c. $9\frac{2}{3}$ ----------

7. Amy bought 12 bottles of fruit juice for her ballet class. If there are 7 people in her ballet class, and the juice is shared out evenly, then how many bottles of juice does each person get? Give your answer as a mixed number.

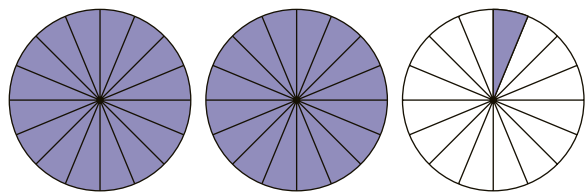
----

8. In a bag of marbles, the ratio of yellow to red marbles is $\frac{12}{11}$, and the ratio of yellow to blue marbles is $\frac{12}{5}$. Express these ratios as mixed numbers.

$\frac{12}{11}$ = ----------

$\frac{12}{5}$ = ----------

9. At the start of lunch, a pizza restaurant had the slices shown on the right. During lunch, 8 slices were eaten. How many pizzas were left? Give your answer as a mixed number.



$\frac{16}{16}$ $\frac{16}{16}$ $\frac{1}{16}$

----

10. A worker added two zippers to a new couch cover. One zipper was $\frac{61}{12}$ feet long and the other was $\frac{5}{2}$ feet long. By converting the lengths into mixed numbers, say which zipper is longer.

Name _____ Class _____ Date _____

Lesson 3.1.3

More on Fractions

California Standard: Number Sense 1.1

1. Convert each of the following to mixed numbers.

a. $-\frac{23}{6}$ ----------

b. $-\frac{29}{6}$ ----------

2. Convert each of the following to improper fractions.

a. $-6\frac{2}{5}$ ----------

b. $-8\frac{5}{6}$ ----------

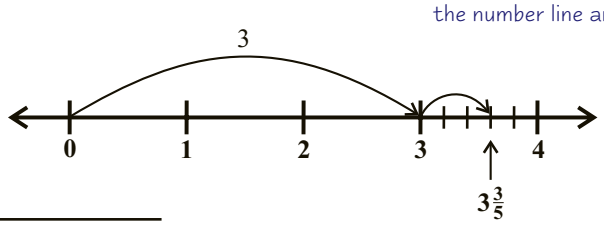
Example

Place $\frac{18}{5}$ on the number line.

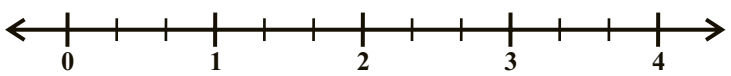
Solution

$\frac{18}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{3}{5} = 3\frac{3}{5}$, so...

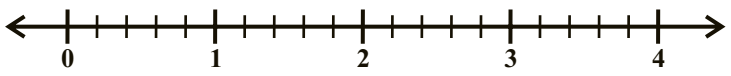
Convert to a mixed number, then place it on the number line and check.



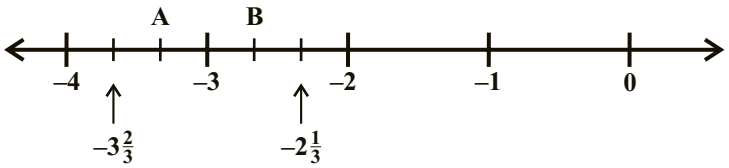
3. Place $\frac{11}{3}$ on this number line.



4. Place $\frac{12}{5}$ on this number line.



5. Which numbers are at positions A and B on this number line?



A: .....

B: .....

6. Juwan said that $-2\frac{1}{5}$ must be further away from 0 than $2\frac{1}{5}$ is, because it has a negative sign. Is Juwan correct? Explain your answer.

.....

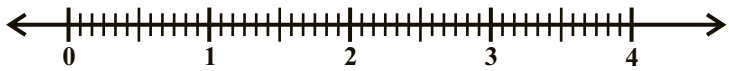
.....

7. Yolanda said that $-\frac{16}{3}$ has the same value as $\frac{-16}{-3}$. Is she correct? Explain your answer.

.....

.....

8. Sharon used $3\frac{1}{6}$ loaves of white bread and $2\frac{1}{4}$ loaves of raisin bread at a French toast breakfast at her school. Put these numbers on the number line below.



9. What number is exactly halfway between $5\frac{1}{3}$ and 8? .....

10. On a very hot day, Kevin was put in charge of distributing water bottles to everyone in school. He gave 29 bottles of water to the first class he visited. If the bottles of water came in boxes of 24, then how many boxes did Kevin give to the first class? Express your answer as a mixed number.

.....

Name _____ Class _____ Date _____

Lesson 3.1.4

Fractions and Decimals

California Standard: Number Sense 1.1

1. Express the following fractions of a dollar as decimals.

a. A dime. ----------

b. A quarter. ----------

c. A half-dollar. ----------

d. A nickel. ----------

2. Convert each fraction to a decimal using the information provided.

a. $\frac{1}{4} = 0.25$ $\frac{2}{4} =$ ---------- $\frac{3}{4} =$ ---------- $\frac{4}{4} =$ ----------

b. $\frac{1}{8} = 0.125$ $\frac{2}{8} =$ ---------- $\frac{5}{8} =$ ---------- $\frac{10}{8} =$ ----------

c. $\frac{1}{5} = 0.2$ $\frac{3}{5} =$ ---------- $\frac{5}{5} =$ ---------- $\frac{10}{5} =$ ----------

3. Find the decimal equivalents of each of the following:

a. $-\frac{3}{8}$ **b.** $-\frac{6}{8}$ **c.** $-\frac{2}{5}$ **d.** $-\frac{4}{5}$

---------- ---------- ---------- ----------

4. Match each fraction to its decimal equivalent by writing the correct letter in each gap.

A: 2.4 B: 0.4 C: –3.75 D: $-0.66\overline{6}$ E: $0.33\overline{3}$

FRACTION	DECIMAL
$\frac{2}{5}$	----------
$\frac{3}{9}$	----------
$-\frac{4}{6}$	----------
$\frac{12}{5}$	----------
$-\frac{15}{4}$	----------

5. Convert each fraction to a decimal.

a. $\frac{3}{4}$ **b.** $-\frac{5}{8}$ **c.** $\frac{12}{9}$ **d.** $\frac{21}{7}$ **e.** $-\frac{3}{11}$ **f.** $\frac{19}{6}$

---------- ---------- ---------- ---------- ---------- ----------

6. Put each list of numbers in order, from least to greatest.

a. 0.75, $-\frac{2}{3}$, –0.8, $\frac{1}{2}$ ----------

b. $-\frac{3}{8}$, $\frac{2}{3}$, –0.8, $\frac{3}{7}$ ----------

Example

James ate 0.37 of a banana. How much of the banana did he eat as a fraction?

Solution

0.37 means "thirty-seven hundredths," so as a fraction it is $\frac{\mathbf{37}}{\mathbf{100}}$.

7. Convert each decimal to a fraction.

a. 0.7 **b.** 0.39 **c.** –0.589 **d.** 0.4539 **e.** 2.7 **f.** –3.43

---------- ---------- ---------- ---------- ---------- ----------

8. Precious has 6 hours in which to do 4 pieces of homework. She plans to spend the same amount of time on each piece of homework. How many minutes will she spend on each piece? Give your answer as a decimal.

----

9. Nine bottles of fruit juice were bought for a party. Each bottle was shared equally between 8 cups. If 15 people attended the party, and each had the same amount of juice, then how many cups did each person get? Give your answer as a decimal.

----

10. Crystal went to the stationery store to get some paper for herself and two friends. Crystal bought 80 sheets of paper, and each of the three friends paid the same amount. How many sheets did each person pay for? Give your answer as a decimal.

Name _____ Class _____ Date _____

Lesson 3.2.1

Multiplying Fractions by Integers

California Standard: Number Sense 2.1

Example

What is $4 \times \frac{3}{5}$?

Solution

To multiply a fraction by an integer, multiply the numerator by the integer and leave the denominator alone. So $4 \times \frac{3}{5} = \frac{4 \times 3}{5} = \frac{\mathbf{12}}{\mathbf{5}}$.

1. Evaluate each of the following multiplications.

a. $4 \times \frac{2}{3}$ ---------- **b.** $5 \times \frac{5}{6}$ ---------- **c.** $4 \times \frac{2}{5}$ ---------- **d.** $5 \times \frac{2}{3}$ ----------

2. Complete each sentence by writing "negative" or "positive" in each blank space.

a. Multiplying a positive fraction by a negative whole number gives a _____ result.

b. Multiplying a negative whole number by a negative fraction gives a _____ result.

c. Multiplying a negative fraction by a positive fraction gives a _____ result.

3. Evaluate each of the following multiplications. Give your answers as mixed numbers.

a. $6 \times \frac{4}{5}$ ---------- **b.** $-\frac{2}{7} \times 4$ ---------- **c.** $-7 \times \frac{3}{11}$ ---------- **d.** $7 \times \frac{5}{6}$ ----------

4. Joe found penlights on sale for $\frac{2}{3}$ of a dollar each. He has $5.00.

How many more dollars would Joe need if he wanted to buy 8 penlights?

---

5. Kaneesha said, "I weigh $\frac{3}{4}$ as much as my mother." If Kaneesha's mother weighs 120 lb, how much does Kaneesha weigh?

---

6. Evaluate each of the following multiplications. Give your answers as mixed numbers.

a. $3 \times \frac{3}{7} =$ ---

b. $5 \times \frac{3}{4} =$ ---

c. $-7 \times \frac{9}{10} =$ ---

Example

What is $3\frac{4}{5} \times 4$?

Solution

$3\frac{4}{5}$ as an improper fraction is $\frac{(3\times5)+4}{5} = \frac{19}{5}$. So $3\frac{4}{5} \times 4 = \frac{19}{5} \times 4 = \frac{19\times4}{5} = \mathbf{\frac{76}{5}}$**, or** $\mathbf{15\frac{1}{5}}$.

7. Evaluate each of the following multiplications. Give your answers as integers or mixed numbers.

a. $5 \times 3\frac{3}{7}$ ---

b. $3 \times 5\frac{2}{3}$ ---

c. $6\frac{3}{4} \times 4$ ---

d. $3\frac{9}{10} \times 3$ ---

8. Evaluate each of the following multiplications. Give your answers as integers or mixed numbers.

a. $-4 \times 2\frac{1}{2}$ ---

b. $-3\frac{2}{3} \times 5$ ---

c. $-2\frac{2}{5} \times (-6)$ ---

d. $-3 \times (-5\frac{1}{4})$ ---

9. Mrs. Jones decides to buy 7 bags of sprouts. Each bag weighs $\frac{3}{4}$ lb.
What weight of sprouts is she going to buy in total? ---

Name _____ Class _____ Date _____

Lesson 3.2.2

More on Multiplying Fractions by Integers

California Standards: Number Sense 2.1, 2.2

1. Mr. Wilson has a set of books, each $1\frac{1}{3}$ inches wide. How wide would a shelf need to be to hold 12 of Mr. Wilson's books?

---

2. Mrs. Martinez buys 12 boards to make a fence. Each board is $3\frac{3}{5}$ ft long. If she uses all 12 boards, how long will the fence be?

---

3. Connor is building a wall. He builds the wall $1\frac{3}{5}$ ft higher per day. In 7 days, how high will the wall be?

---

4. What is $7 \times 5\frac{3}{5}$? _____

5. Mr. Lopez is building a new deck around his house. The builder told Mr. Lopez that it would cost $\$17\frac{1}{2}$ per yard, and that he will build 8 yards. How much will Mr. Lopez pay for his deck?

---

6. Write expressions for each of the following statements.
The first one has been done for you.

a. two-thirds of a number n $\frac{2}{3}n$

b. three-fifths more than a number n _____

c. three-fifths less than a number n _____

d. three-fifths times a number n _____

e. a number, n, added to three-fourths that number _____

f. seven-eighths more than two-thirds of a number n _____

7. Find the area of a rectangle with a length of $7\frac{3}{5}$ ft and a width of 4 ft. Show your work.

_____

_____

_____

_____

8. Find the area of a triangle with a base of 8 inches and a height of 7 inches. Show your work.

_____

_____

_____

_____

9. In Mary Jo's home town, there are two tall buildings. One is 300 ft tall, and the other is $1\frac{1}{3}$ times as tall. How tall is the second building? _____

10. In her home economics class, Tiffany made 36 bran muffins. Two-thirds of them were eaten by the end of the day. How many bran muffins did Tiffany have left to take home? _____

Name _____ Class _____ Date _____

Lesson 3.2.3

Multiplying Fractions by Fractions

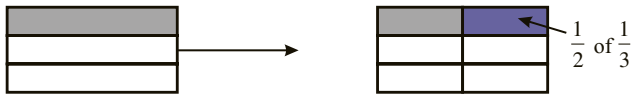
California Standards: Number Sense 2.1, 2.2

Example

What is $\frac{1}{2}$ of $\frac{1}{3}$?

Solution

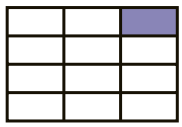
Shade $\frac{1}{3}$ of a rectangle. Shade $\frac{1}{2}$ of this $\frac{1}{3}$ a different color.



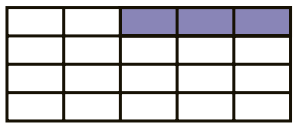
1 part out of 6 is shaded, so $\frac{1}{2}$ of $\frac{1}{3}$ is $\mathbf{\frac{1}{6}}$.

1. What fraction-multiplication problems do these diagrams represent? The first one has been done for you.

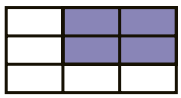
a



b



c



a. $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ **b.** _____ **c.** _____

2. What is $\frac{4}{7} \times \frac{5}{9}$? _____

3. Evaluate each of the following multiplications.

a. $\frac{4}{3} \times \frac{2}{3}$ _____ **b.** $\frac{5}{3} \times \frac{5}{7}$ _____ **c.** $\frac{5}{8} \times \frac{11}{9}$ _____ **d.** $\frac{2}{5} \times \frac{4}{7}$ _____

4. Evaluate each of the following multiplications.

a. $-\frac{2}{7} \times \frac{3}{5}$ _____ **b.** $\frac{5}{6} \times (-\frac{11}{7})$ _____ **c.** $-\frac{3}{4} \times (-\frac{5}{4})$ _____ **d.** $\frac{4}{7} \times \frac{3}{11}$ _____

Example

What is the area of a square with a side length of $1\frac{1}{5}$ inches?

Solution

Area of a square = side length × side length $= 1\frac{1}{5} \times 1\frac{1}{5}$ square inches.

$= \frac{6}{5} \times \frac{6}{5} = \frac{6 \times 6}{5 \times 5} = \frac{36}{25} = \mathbf{1\frac{11}{25}}$ **square inches**.

5. Evaluate each of the following. Give your answers as mixed numbers.

a. $1\frac{2}{5} \times 3\frac{1}{4}$ ---------- **b.** $4\frac{3}{10} \times 2\frac{3}{7}$ ---------- **c.** $5\frac{1}{3} \times 4\frac{6}{7}$ ---------- **d.** $-6\frac{1}{2} \times 4\frac{3}{5}$ ----------

6. Emily has $9\frac{1}{4}$ dollars. Danielle has $3\frac{2}{3}$ times as much as Emily. How much does Danielle have?

---

7. A recipe calls for $2\frac{1}{2}$ cups of flour. Miguel plans to use $3\frac{3}{4}$ times as much of each ingredient as in the recipe. How much flour will Miguel need?

---

8. Evaluate each of the following expressions when $a = \frac{2}{13}$, $b = \frac{1}{11}$, $c = \frac{3}{5}$, and $d = -\frac{7}{5}$.

a. abd ----------

b. $b \times d$ ----------

c. $cabd$ ----------

9. At a party, Angel ate $\frac{2}{5}$ of the rye bread. Kayla then ate $\frac{1}{2}$ of what was left. If no one else at the party ate any rye bread, what fraction of the rye bread did Kayla eat?

Name _____ Class _____ Date _____

Lesson 3.3.1

Dividing by Fractions

California Standards: Number Sense 2.1, 2.2

Example

What is the reciprocal of $2\frac{3}{7}$?

Solution

$2\frac{3}{7}$ as an improper fraction is $\frac{17}{7}$, so the reciprocal of $2\frac{3}{7}$ is $\mathbf{\frac{7}{17}}$.

1. Find the reciprocal of each of the numbers below.

a. $\frac{2}{3}$ ------ **b.** $\frac{7}{11}$ ------ **c.** $\frac{4}{17}$ ------ **d.** $2\frac{2}{3}$ ------ **e.** $4\frac{6}{7}$ ------ **f.** $7\frac{2}{5}$ ------

Example

What is $\frac{3}{5} \div \frac{1}{2}$?

Solution

$\frac{3}{5} \div \frac{1}{2} = \frac{3}{5} \times \frac{2}{1} = \frac{3 \times 2}{5 \times 1} = \mathbf{\frac{6}{5}}$.

2. **a.** Rewrite $\frac{3}{4} \div 5$ as a multiplication.

---

b. Evaluate $\frac{3}{4} \div 5$.

---

3. What is $\frac{3}{4} \div 2$? ---

4. Evaluate each of the following divisions.

a. $\frac{3}{4} \div 5 =$ ------ **b.** $\frac{4}{9} \div 7 =$ ------ **c.** $\frac{7}{11} \div 9 =$ ------ **d.** $3\frac{4}{5} \div 4 =$ ------

5. How many sixths are there in $\frac{2}{3}$?

---

6. Jan intends to paint $\frac{1}{2}$ a room in three days. How much of the room will she paint each day?

---

7. Calculate each of the following.

a. $\frac{3}{4} \div \frac{2}{3} =$

---

b. $\frac{7}{5} \div \frac{3}{4} =$

---

c. $\frac{4}{5} \div \frac{7}{9} =$

---

d. $\frac{9}{13} \div \frac{5}{7} =$

---

e. $\frac{3}{5} \div \frac{3}{10} =$

---

8. What is $\frac{57}{67} \div \frac{57}{67}$? What do you get when you divide any number by itself?

---

9. Joan divided $\frac{1}{5}$ by a number, and got an answer of $\frac{1}{15}$. What number did she divide by?

---

10. Jesus divided a piece of rope $6\frac{2}{3}$ ft long into 3 equal lengths. How long is each length? Give your answer as a mixed number.

---

11. Mark has $7\frac{3}{4}$ lb of modeling clay for a class of 30 people. How much clay will each person get?

Name _____ Class _____ Date _____

Lesson 3.3.2

Solving Problems by Dividing Fractions

California Standard: Number Sense 2.1

Example

Alexis has $1\frac{1}{2}$ cartons of orange juice. She gives half to Zachary. How many cartons of juice does she have left?

Solution

Alexis is left with $1\frac{1}{2} \div 2$ cartons of juice.

$1\frac{1}{2} = \frac{3}{2}$, so $1\frac{1}{2} \div 2 = \frac{3}{2} \div 2 = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$. **Alexis has $\frac{3}{4}$ of a carton left.**

1. The day after a party, Leandra invited 4 of her friends over to help finish $\frac{2}{3}$ of a bowl of raisins. If Leandra and her friends each had the same amount, what fraction of a bowl did they each eat?

---

2. What operation could you use to split a number into equal parts?

---

3. To find $\frac{3}{4}$ of 8, Jacob calculated $\frac{3}{4} \times \frac{1}{8}$. He got an answer of $\frac{3}{32}$. Did Jacob find the correct answer? Explain your answer.

---

4. Mrs. Baxter bought $8\frac{3}{4}$ cubic yards of red rock to landscape her yard. She will use a wheelbarrow that holds $\frac{2}{3}$ cubic yards to spread the red rock. How many wheelbarrow trips will be required?

Example

Jazmin cooks 2 ounces of wholegrain rice and $3\frac{3}{4}$ ounces of basmati rice. She intends to share the rice evenly between herself and two friends. How much rice will each friend get altogether?

Solution

In total, Jazmin has cooked $2 + 3\frac{3}{4} = 5\frac{3}{4}$ ounces of rice.
She shares the rice between a total of 3 people.
So each friend gets $5\frac{3}{4} \div 3 = \frac{23}{4} \times \frac{1}{3} = \frac{23}{12} = \mathbf{1\frac{11}{12}}$ **ounces**.

5. Miles had 3 boxes of red pencils and $4\frac{1}{3}$ boxes of blue pencils. He distributes them so that each of his 9 friends gets the same number of pencils. How many boxes of pencils does each friend get?

---

6. Ross has 3 fifteen-gallon tanks of gasoline. He needs to fill some $\frac{3}{4}$-gallon gasoline tanks for a toy-car show. How many tanks could he fill?

---

7. Sally used $9\frac{1}{8}$ gallons of water to wash 4 mini-bikes. What is the average amount of water that she used on each mini-bike? Give your answer as a mixed number.

---

8. Paul had $\frac{1}{2}$ of a gallon of orange juice. He wanted to fill glasses that could hold $\frac{1}{8}$ of a gallon each. How many glasses could Paul fill?

Name _____ Class _____ Date _____

Lesson 3.4.1

Making Equivalent Fractions

California Standard: Number Sense 2.1

Example

Find a fraction equivalent to $\frac{3}{7}$ that has a denominator of 21.

Solution

$\frac{3}{7} = \frac{3}{7} \times 1 = \frac{3}{7} \times \frac{3}{3} = \frac{3\times3}{7\times3} = \frac{\mathbf{9}}{\mathbf{21}}$.

1. Find 3 fractions equivalent to $\frac{2}{3}$.

---

2. Find the missing number in each equation.

a. $\frac{2}{3} = \frac{x}{12}$ ---

b. $\frac{5}{7} = \frac{y}{63}$ ---

c. $\frac{3}{8} = \frac{12}{z}$ ---

d. $\frac{u}{8} = \frac{27}{72}$ ---

e. $\frac{7}{v} = \frac{28}{36}$ ---

3. Write down all the common factors of 12 and 36.

---

4. Find the missing number in each of the following.

a. $\frac{6}{8} = \frac{m}{4}$ ---

b. $\frac{12}{15} = \frac{n}{5}$ ---

c. $\frac{16}{24} = \frac{2}{p}$ ---

d. $\frac{36}{45} = \frac{4}{q}$ ---

e. $\frac{r}{35} = \frac{3}{5}$ ---

Example

400 students were asked to name their favorite car color. 130 said red. What fraction of the students said red? Simplify your fraction as much as possible.

Solution

130 of 400 students said red. As a fraction, that is $\frac{130}{400}$.

130 and 400 can both be divided by 10, so you can write this as $\frac{130}{400} = \frac{13\times10}{40\times10}$.

Then cancel the common factor of 10: $\frac{130}{400} = \frac{13}{40}$.

13 and 40 have no common factors, so $\mathbf{\frac{13}{40}}$ is in its simplest form.

5. 270 of the 300 people running a local marathon finished. What fraction of the runners finished? Simplify your fraction as much as possible.

---

6. 130 of the 270 students in a school went on a trip to France. What fraction of the students went to France? Simplify your fraction as much as possible.

---

7. Marissa recently bought a scooter, which cost $\frac{3}{50}$ of the \$1250 she had in her bank account. How much did the scooter cost?

---

8. Antonio gave $\frac{1}{50}$ of \$1450 to charity. How much in dollars did Antonio give to charity?

Name _____ Class _____ Date _____

Lesson 3.4.2

Finding the Simplest Form

California Standard: Number Sense 2.4

1. Find the prime factorization of each number below.

a. 60 ----------

b. 90 ----------

Example

What is the greatest common divisor of 24 and 36?

Solution

Find the prime factorizations of 24 and 36.

$24 = 2 \times 2 \times 2 \times 3$
$36 = 2 \times 2 \times 3 \times 3$

(and)

24 and 36 both have factors of 2 (twice), and 3.

So the greatest common divisor of 24 and 36 is $2 \times 2 \times 3 = 12$.

2. Find the greatest common divisor (GCD) of each pair of numbers.

a. 12, 18 ------- **b.** 12, 30 ------- **c.** 30, 40 ------- **d.** 30, 60 ------- **e.** 40, 56 ------- **f.** 40, 60 ------- **g.** 12, 56 -------

3. Freddie is trying to find the simplest form of $\frac{300}{800}$. He has found that 100 is the greatest common divisor of 300 and 800. What should he do next to find the simplest form of the fraction?

---

4. Reduce each fraction to its simplest form.

a. $\frac{12}{18}$ ------- **b.** $\frac{12}{30}$ ------- **c.** $\frac{30}{40}$ ------- **d.** $\frac{30}{60}$ ------- **e.** $\frac{40}{56}$ ------- **f.** $\frac{40}{60}$ ------- **g.** $\frac{36}{90}$ -------

5. There were 24 males and 8 females in a class. What fraction of the students were male, and what fraction were female? Give your answers in their simplest form.

Male: ---------- Female: ----------

6. $\frac{4}{5}$ of the 30 students in a class are girls. How many girls are in the class?

---

7. The table on the right shows the number of adults and children that went to the aquarium on the weekend. Use it to answer the following questions. Give all your answers in their simplest form.

Date	Adults	Children
Saturday	34	47
Sunday	56	63

a. What fraction of the weekend's visitors were adults?

---

b. What fraction of the weekend's visitors were children? ----------

8. Mrs. Salazar's farm has 45 cows and 75 sheep, and no other animals. What fraction of Mrs. Salazar's animals are cows? Give your answer in its simplest form.

---

9. For a pep rally, the football team, the marching band, and the drill team all decide to march together. Each team wants to march as a separate rectangular group, but they decide that all three groups should have the same number of students in each row. If there are 48 members of the football team, 60 members of the marching band, and 36 members of the drill team, then what is the maximum number of students they can put in each row?

Name _____ Class _____ Date _____

Lesson 3.4.3

Fraction Sums

California Standard: Number Sense 2.1

Example

What is $\frac{3}{7} + \frac{2}{7}$?

Solution

$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \mathbf{\frac{5}{7}}$.

1. Work out each of the following.

a. $\frac{5}{9} + \frac{7}{9} + \frac{2}{9} =$ ----------

b. $\frac{12}{40} + \frac{15}{40} - \frac{8}{40} =$ ----------

c. $\frac{50}{70} + \frac{21}{70} - \frac{24}{70} =$ ----------

2. Complete the following sentence by writing either "numerators" or "denominators" in the spaces.

To add fractions that have the same _____ , just add the _____ and keep the _____ the same.

3. Calculate each sum, giving all your answers in their simplest form. Use mixed numbers for answers greater than 1.

a. $\frac{4}{9} + \frac{2}{9}$ ----------

b. $\frac{5}{12} + \frac{4}{12}$ ----------

c. $\frac{3}{5} + \frac{4}{5}$ ----------

d. $\frac{7}{8} + \frac{5}{8}$ ----------

4. Calculate each sum, giving all your answers in their simplest form. Use mixed numbers for answers greater than 1.

a. $\frac{4}{15} + \frac{1}{15}$ ----------

b. $\frac{11}{12} - \frac{7}{12}$ ----------

c. $\frac{5}{12} - \frac{2}{12}$ ----------

d. $\frac{3}{4} + \frac{3}{4}$ ----------

5. Savannah added the fractions $\frac{8}{15}$ and $\frac{7}{15}$, and got the answer $\frac{15}{30}$.
What has she most likely done wrong?

---

6. An oat bar was cut into 8 pieces. Jorge ate $\frac{3}{8}$ of the bar, and Nuria ate $\frac{4}{8}$ of it. How much was left?

---

7. A walking trail is marked with yellow flags for the first $\frac{3}{16}$ of its length, and white flags for the next $\frac{7}{16}$ of its length. What fraction of the trail is left after the end of the white flags?
Give your answer in its simplest form.

---

8. In a recent basketball game, Terell scored $\frac{2}{11}$ of the team's points, and was awarded an assist for another $\frac{1}{11}$ of the points. What fraction of the points did Terell either score or assist?

---

9. At the end of Wednesday, Janell had $\frac{19}{24}$ of a book left to read. She read $\frac{5}{24}$ of it on Thursday, and $\frac{7}{24}$ of it over the weekend. How much of the book did she have left to read after the weekend?

Name _____ Class _____ Date _____

Lesson 3.4.4 Fractions with Different Denominators

California Standards: Number Sense 1.1, 2.1

1. Find a common multiple for each of the following pairs of numbers.

a. 4 and 6 ---------- **b.** 8 and 12 ----------

c. 10 and 14 ---------- **d.** 8 and 15 ----------

Example

What is $\frac{1}{3} + \frac{1}{4}$?

Solution

To add the fractions, first find equivalent fractions with the same denominator.

$\frac{1}{3} = \frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$ $\quad\quad$ $\frac{1}{4} = \frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

So $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \mathbf{\frac{7}{12}}$.

2. Calculate each of the following. Give your answers in their simplest form.

a. $\frac{1}{3} + \frac{1}{2}$ ---------- **b.** $\frac{3}{5} + \frac{1}{10}$ ---------- **c.** $\frac{3}{4} - \frac{5}{8}$ ---------- **d.** $\frac{1}{3} - \frac{5}{6}$ ----------

3. Calculate each of the following. Give your answers in their simplest form. Use mixed numbers for answers greater than 1.

a. $\frac{2}{3} + \frac{3}{4}$ ---------- **b.** $\frac{2}{5} + \frac{4}{7}$ ---------- **c.** $\frac{5}{7} - \frac{3}{4}$ ---------- **d.** $\frac{4}{7} - \frac{2}{3}$ ----------

e. $\frac{4}{5} + \frac{3}{4}$ ---------- **f.** $\frac{2}{7} - \frac{3}{4}$ ---------- **g.** $\frac{1}{6} - \frac{2}{5}$ ---------- **h.** $\frac{6}{7} + \frac{2}{3}$ ----------

4. Calculate each of the following. Give your answers in their simplest form. Use mixed numbers for answers greater than 1.

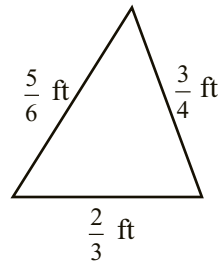
a. $\frac{5}{12} + \frac{4}{9}$ **b.** $\frac{3}{8} - \frac{7}{12}$ **c.** $\frac{4}{15} + \frac{7}{10}$ **d.** $\frac{11}{20} - \frac{11}{12}$

---

5. Andres is knitting a sweater. After knitting $\frac{7}{8}$ in., he notices a mistake and has to undo $\frac{1}{4}$ in. How many inches of the knitting did Andres not undo?

---

6. Find the perimeter of the triangle shown on the right.



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7. Determine the missing number in each of the following:

a. $\frac{2}{3} + \frac{3}{4} - ? = 1$ **b.** $\frac{1}{2} + \frac{1}{3} + ? = 1$ **c.** $\frac{1}{4} + ? + \frac{2}{7} = 1$

---

8. Mary is dressing dolls. She needs $2\frac{1}{3}$ ft of ribbon for one doll, $1\frac{3}{4}$ ft for a second doll, and $3\frac{1}{2}$ ft for a third doll. How much ribbon does she need for all three dolls?

---

9. Kyle worked out $\frac{9}{20}$ of his math problems before he left school, $\frac{1}{5}$ on the bus, and the rest at home. What fraction of the math problems did he do at home?

---

10. Helen mailed $\frac{2}{3}$ of the invites to a party on Tuesday, $\frac{1}{12}$ on Wednesday, and the rest on Thursday. What fraction did she mail on Thursday?

---

11. A computer store took a large shipment of computers, and sold $\frac{23}{30}$ of them in December. They sold $\frac{1}{10}$ of the shipment in January, and $\frac{1}{15}$ in February. What fraction of the shipment is left?

Name _____ Class _____ Date _____

Lesson 3.4.5

Least Common Multiples

California Standards: Number Sense 2.1, 2.4

Example

Find the least common multiple of 24 and 36.

Solution

Multiples of 24: 24, 48, 72, 96, 120, 144, ...
Multiples of 36: 36, 72, 108, ...
The first number to appear in both lists is 72. So 72 is the least common multiple of 24 and 36.

1. List the first 10 multiples of each of the following numbers.

a. 8 ----------

b. 12 ----------

c. 18 ----------

d. 20 ----------

e. 24 ----------

f. 30 ----------

g. 36 ----------

2. Determine the least common multiple of the following pairs of numbers.

Use your answers from Exercise 1 to help.

a. 8, 12 ------- **b.** 12, 18 ------- **c.** 12, 20 ------- **d.** 20, 24 ------- **e.** 30, 36 ------- **f.** 20, 30 ------- **g.** 18, 30 -------

3. Determine the least common multiple of the following sets of numbers.

a. 8, 12, 18 ---------- **b.** 18, 20, 30 ---------- **c.** 12, 20, 24 ---------- **d.** 8, 12, 18, 24 ----------

Find the least common multiple of 3 numbers in the same way as you do it for 2.

4. Find the least common multiple of each pair of numbers.

a. 20 and 15 ---------- **b.** 32 and 48 ----------

5. Hadley is finding the least common multiple of 8 and 18. So far she has written the following:
Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104
Multiples of 18:
At which number can Hadley stop her list of multiples of 18?

---

6. Juan is trying to find the least common multiple of two numbers, and spots that the smaller number is a factor of the bigger number. What can he now say is the least common multiple of the two numbers?

---

7. Veronica has a number of segments of orange. She has calculated that she could make either bags with 3 orange segments each or bags with 5 orange segments each, with no orange segments left over. What is the least number of segments of orange that Veronica could have?

---

8. A bus stops at a particular stop every 6 minutes. A second bus stops at the same stop every 9 minutes. A third bus stops at the same stop every 12 minutes. All three buses are at the stop at 12:00 noon. When will all three buses next be at the stop together?

---

9. Race car #22 can go around the track in 4 minutes. Race car #33 can go around the track in 3 minutes. The cars start the race side-by-side at 1:00 p.m. At what time will the cars next be side-by-side while crossing the start line?

Name _____ Class _____ Date _____

Lesson 3.4.6

Mixed Numbers and Word Questions

California Standards: Number Sense 2.1, 2.4

1. Which of these fractions is greater: $\frac{3}{4}$ or $\frac{2}{3}$?

Explain how you made your decision. Do not convert the fractions to decimals.

---

2. Calculate each of the following. Give your answers as mixed numbers.

a. $7 + 2\frac{3}{4}$ **b.** $5\frac{2}{3} - 3$ **c.** $3\frac{4}{5} + 5$ **d.** $7\frac{5}{9} - 9$ **e.** $5 - 2\frac{5}{6}$

---------- ---------- ---------- ---------- ----------

Example

What is $2\frac{3}{4} + 3\frac{1}{5}$?

Solution

First turn these mixed numbers into improper fractions: $2\frac{3}{4} = \frac{11}{4}$ and $3\frac{1}{5} = \frac{16}{5}$.

To add $\frac{11}{4}$ and $\frac{16}{5}$, you need to find fractions equivalent to each of these that have a common denominator.

$\frac{11}{4} = \frac{11}{4} \times \frac{5}{5} = \frac{55}{20}$ and $\frac{16}{5} = \frac{16}{5} \times \frac{4}{4} = \frac{64}{20}$.

So $2\frac{3}{4} + 3\frac{1}{5} = \frac{11}{4} + \frac{16}{5} = \frac{55}{20} + \frac{64}{20} = \mathbf{\frac{119}{20}}$.

3. Calculate each of the following. Give your answers as mixed numbers.

a. $4\frac{2}{3} + 3\frac{4}{5}$ **b.** $5\frac{5}{6} - 2\frac{4}{5}$ **c.** $2\frac{3}{4} - 4\frac{5}{6}$ **d.** $6\frac{5}{8} - 2\frac{1}{6}$ **e.** $3\frac{5}{12} + 4\frac{7}{8}$

---------- ---------- ---------- ---------- ----------

4. Nicole wrote the following:

$$1\frac{3}{5} \times 2\frac{4}{5} = \frac{13}{5} \times \frac{24}{5} = \frac{312}{25}$$

What mistake did Nicole make?

---

5. Levar has $8\frac{3}{4}$ pounds of potatoes. He plans to use 6 pounds for a dinner party.
How many pounds will Levar have left?

---

6. One morning, the snow level on Mount Baker was $4\frac{1}{4}$ feet. In the afternoon, $2\frac{5}{6}$ feet of snow fell.
What was the snow level at the end of the day?

---

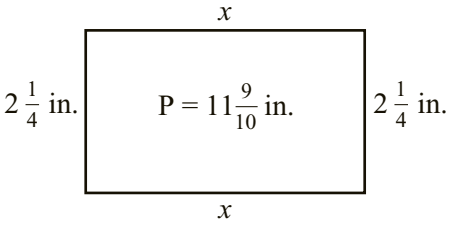
7. Yesenia knitted $8\frac{2}{3}$ inches of a sweater. She then noticed an error, so she unpicked $2\frac{3}{4}$ inches.
How many inches of sweater were left?

---

8. Hector is a football player, so he carefully watches his weight. One month ago, he weighed $120\frac{4}{5}$ lb. Today, he weighs $124\frac{3}{4}$ lb. How much weight did he gain during the month?

---

9. The perimeter of the rectangle shown below is $11\frac{9}{10}$ in. What is the unknown length, x?



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10. A 30-gallon water tank has $17\frac{3}{4}$ gallons of water in it. How many more gallons of water are needed to fill the tank?

Name _____ Class _____ Date _____

Lesson 3.5.1

Fractions and Percents

California Standard: Number Sense 1.4

Example

R% as a fraction is $\frac{37}{100}$. What is the value of R?

Solution

% just means "out of 100," so $\frac{37}{100}$ as a percent is 37%. So $\boldsymbol{R = 37}$.

1. Write each fraction as a percent.

a. $\frac{57}{100}$ **b.** $\frac{21}{100}$ **c.** $\frac{87}{100}$ **d.** $\frac{9}{100}$ **e.** $\frac{50}{100}$ **f.** $\frac{63}{100}$

_____ _____ _____ _____ _____ _____

2. Write each percent as a fraction in its simplest form.

a. 47% **b.** 40% **c.** 65% **d.** 129% **e.** 287% **f.** $33\frac{1}{3}$%

_____ _____ _____ _____ _____ _____

3. What is $\frac{3}{4}$ as a percent?

_____

4. Write each fraction as a percent.

a. $\frac{3}{20}$ **b.** $\frac{7}{25}$ **c.** $\frac{9}{40}$ **d.** $\frac{64}{50}$ **e.** $\frac{7}{4}$ **f.** $\frac{16}{10}$

_____ _____ _____ _____ _____ _____

Example

Daisy has played on the football team 40 times this year. She has caught passes in 35 of those games. In what percent of the games that she's played this year has Daisy caught a pass?

Solution

Daisy has caught a pass in $35 \div 40 = 0.875$ of her games.
To convert this to a percent, multiply by 100: $0.875 = 0.875 \times 100\% =$ **87.5%.**

5. Gabriel has 50 baseball cards, of which 21 are pitchers.
What percent of Gabriel's baseball cards are pitchers?

---

6. Mr. Lucas has 25 coins in his pocket, of which 11 are quarters.
What percent of the coins are quarters?

---

7. In a field, there are 24 horses and 36 cows. What percent of those animals are horses?

---

8. Of the 75 students interviewed, 25 said the hot lunch was good today.
What percent of the students interviewed liked the hot lunch? ---

9. The instruments owned by the marching band are 34% trombones, 24% drums, and 42% saxophones. If the band owns 50 instruments in total, how many of each musical instrument are there?

a. Trombones ---

b. Drums ---

c. Saxophones ---

10. Erin is cleaning windows on a building. She cleaned 80 of the 250 windows in the morning, and another 40 in the afternoon.

a. What percent of the windows has Erin cleaned so far? ---

b. What percent of the windows are left to clean? ---

Name _____ Class _____ Date _____

Lesson 3.5.2

Percents and Decimals

California Standard: Number Sense 1.4

Example

What is 22% as a decimal?

Solution

$22\% = \frac{22}{100} = \mathbf{0.22}$.

1. Convert each of the following to decimals.

a. 39% **b.** 57% **c.** 47% **d.** 137% **e.** 245% **f.** 75%

_____ _____ _____ _____ _____ _____

2. Convert each of the following to percents.

a. 0.56 **b.** 0.99 **c.** 1.07 **d.** 1.47 **e.** 3.58 **f.** 0.4

_____ _____ _____ _____ _____ _____

3. Convert each of the following fractions to percents.

a. $\frac{1}{5}$ **b.** $\frac{1}{4}$ **c.** $\frac{1}{3}$ **d.** $\frac{1}{2}$ **e.** $\frac{3}{4}$ **f.** 1

_____ _____ _____ _____ _____ _____

4. Use the given fact to do each conversion below.

a. $\frac{1}{5} = 20\%$. What is $\frac{2}{5}$ as a percent?

_____

b. $\frac{1}{3} = 33\frac{1}{3}\%$. What is $\frac{2}{3}$ as a percent?

_____

c. $\frac{1}{2} = 50\%$. What is $\frac{2}{2}$ as a percent?

_____

d. $\frac{1}{5} = 20\%$. What is $\frac{4}{5}$ as a percent?

_____

5. Order 43%, $\frac{2}{5}$, and 0.3 from least to greatest.

---

6. Order 88%, $\frac{7}{8}$, and 0.9 from least to greatest.

---

7. Of the 12 pencils in a box, 4 are yellow. What percent of the pencils are yellow?

---

8. Twenty-five players are on a team, and in one game, 15 of them played.
What percent of the players played?

---

9. 120 people tried out for the football team, but Coach Jacobs was only looking for 75 players.

a. What percent of people trying out could make the team? ---

b. What percent of people trying out cannot make the team? ---

10. A construction worker had 135 white bricks and 35 tan bricks to use for the front of a new building. Give your answer to each of the following questions to the nearest tenth of a percent.

a. What percent of the bricks were white? ---

b. What percent of the bricks were tan? ---

c. 27 white bricks were left over after the front of the building had been completed. What percent of the white bricks were not used?

Name _____ Class _____ Date _____

Lesson 3.5.3

Percents of Numbers

California Standard: Number Sense 1.4

Example

What is 20% of 50?

Solution

20% as a decimal is 0.2. So 20% of 50 = 0.2 × 50 = **10.**

1. Sarah needed to find 80% of 27. She first changed 80% to the decimal 0.80. What should Sarah do next to find 80% of 27?

---

2. Find each of the following:

a. 30% of 60 ---

b. 45% of 80 ---

c. 60% of 20 ---

d. 130% of 70 ---

3. Miguel is using a shortcut to find 60% of 200. He knows that 10% of 200 is 20. What must he multiply 20 by to get 60% of 200?

---

4. In a survey of Jonathon's class of 35 students, 80% said they liked the hot-lunch program. How many students liked the hot-lunch program?

---

5. Order the following numbers from least to greatest: $42\%, \frac{2}{5}, 0.41, \frac{1.17}{3}$

---

6. Of 200 sixth graders, 85% scored 6.3 or higher in a mathematics test. How many students scored 6.3 or higher?

---

7. Of 50 teachers, only 12% do not exercise. What number of teachers do exercise?

---

8. Of the 140 tickets sold for the school play, 20% were sold to students, 75% to relatives of students, and the remaining 5% to other people.

a. What number of tickets were sold to students? ---

b. What number of tickets were sold to relatives of students? ---

c. What number of tickets were sold to other people? ---

9. Mrs. Juarez sold 180 frames at her shop last month. 45% of them were for photographs, 35% were for paintings, and the rest were for miscellaneous items.

a. What number of frames were for photographs? ---

b. What number of frames were for paintings? ---

c. What number of frames were for miscellaneous items? ---

10. The total bill at a restaurant was $43.20. Jake said he would pay 75% of the bill, because his family had eaten 75% of the food. The other 25% was paid by Mario.

a. How much money did Jake pay? ---

b. How much money did Mario pay? ---

Name _____ Class _____ Date _____

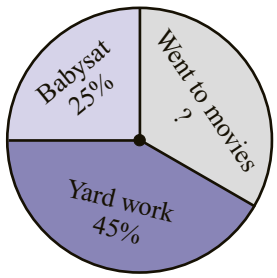
Lesson 3.5.4

Circle Graphs and Percents

California Standard: Number Sense 1.4

Example

A group of 6th grade students were asked what they did over the past weekend. This circle graph shows the results of the survey. What percent of the students went to the movies?



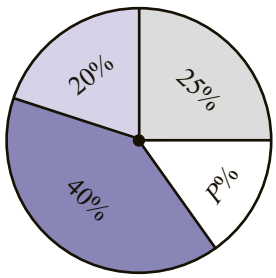
Solution

The whole circle always represents 100%.
So call the percent of students that went to the movies m%.

Then
$$\begin{aligned} 25 + 45 + m &= 100 \\ 70 + m &= 100 \\ m &= 100 - 70 = 30 \end{aligned}$$

So **30%** of the students in the survey went to the movies over the past weekend.

1. What is the value of P on the circle graph on the right?



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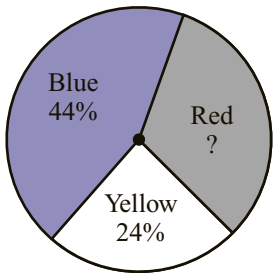
2. In a survey, people were asked which of three colors was their favorite. The results of the survey are shown on the circle graph on the right. What percent of people who answered the survey said they preferred red?



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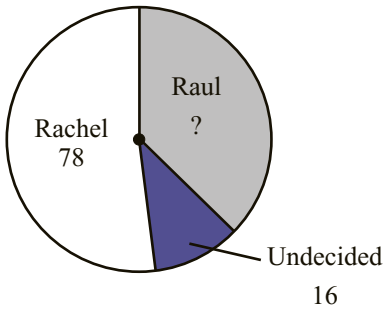
3. A group of 150 students were randomly selected and asked who they would choose for student body president. The results are shown on the circle graph on the right. How many students said they would choose Raul?



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4. A group of students were asked which of four meal options they preferred. The circle graph below shows the number of students who said they preferred each option.

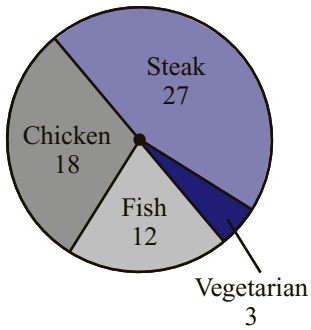
a. How many students were questioned in the survey? ---

b. What percent of the group preferred steak? ---

c. What percent of the group preferred chicken? ---

d. What percent of the group preferred fish? ---

e. What percent of the group preferred vegetarian food? ---



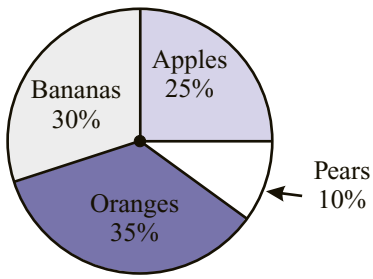
5. 500 students were surveyed about their favorite fruit. The circle graph on the right shows the results. What number of students chose each fruit?

a. Oranges ---

b. Bananas ---

c. Apples ---

d. Pears ---



6. The age groups of the members of a recycling club are displayed in the circle graph below.

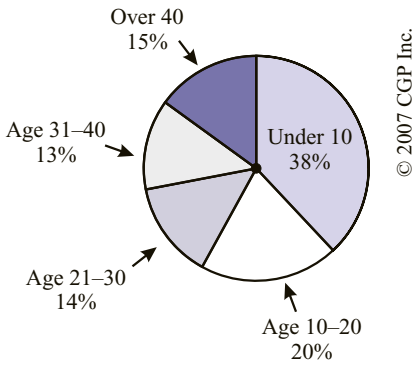
a. What is the total number of people in the club, if there are 60 people aged from 10 to 20? ---

b. What percent of people are aged from 21 to 40? ---

c. How many people are aged from 21 to 40? ---

d. What percent of people are 30 years of age or younger? ---

e. How many people are 30 years of age or younger? ---



Name _____ Class _____ Date _____

Lesson 3.5.5

Percent Increase

California Standard: Number Sense 1.4

Example

Mr. Guzman took his family out to dinner. The bill was $146. He wants to tip 15%. How much should he tip?

Solution

Mr. Guzman should tip 15% of $146 = 0.15 × 146 = **$21.90**.

1. **a.** What is 176 increased by 10%? _____

b. What is 124 increased by 25%? _____

c. What is 254 increased by 84%? _____

2. Nisha took her friend to lunch. The bill was $21.45. Nisha decided to tip 18%, and rounded her tip to the nearest whole cent. How much did Nisha leave for the tip?

_____

3. Adam bought a new shirt with a price tag of $18.00. There was a sales tax of 8.5% added to the price of the shirt. What did Adam pay for the shirt, including the tax?

_____

4. Mrs. Brown took her children to lunch. The bill was $14.37. She decided to tip 20%. What was the amount of the tip, rounded to the nearest whole dollar?

_____

Example

Ralph invested $1000.00 of his college savings. After one year, the fund was worth $1150.00. What was the percent increase?

Solution

The fund went up by 1150 – 1000 = $150.
The initial investment was $1000, and 150 ÷ 1000 = 0.15, which is equivalent to 15%.
So the percent increase was **15%.**

5. Mrs. Garcia had a piano moved. The movers charged $800.00. She gave them a check for $1000.00, which included a tip. What percent did Mrs. Garcia tip?

---

6. Cameron went to the local newsstand and spent $4.37. He decided to leave a tip and paid $6.00. What percent tip did he leave? Round your answer to the nearest whole percent.

---

7. Ms. Smith went to the beauty shop and had a bill of $66.75. She gave the shop $80. What percent tip did Ms. Smith leave? Round your answer to the nearest tenth of a percent.

---

8. Tahnee found a new outfit for $50.00. There is an 8.4% sales tax. How much did Tahnee have to pay?

---

9. Cesar and his girlfriend went to dinner. The bill was $42.38. His girlfriend decided to pay a 15% tip, and left $6.50 in addition to the cost of the meal. Did she leave enough for a 15% tip?

---

10. After getting a lunch bill of $38.94, Mr. Gerhart left $45.00 for the bill and the tip. What percent did Mr. Gerhart tip?

---

11. Monique invested $2000.00. After one year, the investment was worth $2100.00. What was the percent increase of the investment?

Name _____ Class _____ Date _____

Lesson 3.5.6

Percent Decrease

California Standard: Number Sense 1.4

1. Calculate the amount left after each decrease.

a. 48 decreased by 25% _____

b. 120 decreased by 90% _____

c. 190 decreased by 48% _____

d. 150 decreased by 35% _____

Example

At the end of the day, a supermarket applies a 60% discount to all unsold bakery goods. How much would a loaf of bread that had been \$1.50 cost at the end of the day?

Solution

The bread has been reduced by 60%.
The reduction is $0.6 \times \$1.50 = \$0.90$.
So the reduced price is $\$1.50 - \$0.90 =$ **\$0.60**, or **60¢**.

2. A men's clothing store is having a sale. Every item in the store is discounted by 40%. Mr. Lopez finds a shirt that has a price tag of \$35.00. How much is the shirt with the discount?

_____

3. A department store was offering 30% off all items. Bryan bought a pair of shoes which had a price tag of \$70.00. How much were the shoes with the discount?

_____

Example

A pair of jeans should have cost \$50, but they were reduced to \$30 in a sale.
What percent decrease does this represent?

Solution

The jeans have been discounted by 50 – 30 = \$20.
So, if the percent decrease is p then:

$$50 \times p = 20$$
$$p = 20 \div 50$$
$$= 0.4 = 40\%.$$

The discount was 40%.

4. Joseph invested \$2000.00. After one year, his investment was worth \$1600.00. What was the percent decrease of Joseph's investment?

---

5. A women's clothing store offered 30% off all pants. After two weeks, it announced that it was reducing the prices by a further 14%. Anna found a pair of pants with a tag price of \$45.00. How many dollars less than the original price would the pants be selling for?

---

6. The number of people visiting a chain of cinemas went up by 25% last week. This week, it went down by 15%. If there was an average of 2000 daily customers originally, how many daily customers were there, on average, this week?

---

7. A department store reduced the price of all blouses by 30%. Mildred bought a blouse with a tag price of \$27.00. How much did Mildred pay for the blouse, including a 6.5% sales tax?

---

8. The number 80 is decreased to 70. What was the percent decrease?

Name _____ Class _____ Date _____

Lesson 3.5.7

Simple Interest

California Standard: Number Sense 1.4

1. Explain what each variable in the formula $I = Prt$ represents.

Example

Mr. Perez has a \$5000.00 bond which will pay 7% yearly simple interest for 5 years. How much will his investment be worth after 5 years?

Solution

$I = Prt$ $P = \$5000$

$r = 7\% = 0.07$

$t = 5$ years

So $I = 5000 \times 0.07 \times 5 = \$1750$. The investment will have increased by \$1750, so it will be worth $5000 + 1750 = \$6750$.

2. Laura invested \$1000.00 in an account that paid 6% simple interest. What was her investment worth after one year?

---

3. William put \$400.00 in a bank savings account that pays 5.3% simple interest per year. He decided to withdraw his money and close the account after 3 years. How much money did he withdraw?

---

4. Jorge invested \$500.00 in a simple interest fund. At the end of 5 years, his investment was worth \$725.00. What was his yearly rate of interest?

---

5. Jeremy took out a $50,000, 10-year loan with a simple interest rate of 6% per year.

a. What is the total amount of interest Jeremy will have to pay on the loan?

---

b. How much interest would Jeremy have had to pay if the interest rate had been 8%?

---

c. How much interest would Jeremy pay if the interest rate had been 8% and the principal had been 20% larger?

Example

Amber has $900 in a bank account that pays 4% yearly simple interest.
How much interest does she earn a month?

Solution

$I = Prt$ $\quad P = \$900$

$r = 4\% = 0.04$

$t = 1 \text{ month} = \frac{1}{12} \text{ year.}$

So $I = 900 \times 0.04 \times \frac{1}{12} = \$3$. **Amber earns $3 interest a month.**

6. Maria borrowed $3000, at a simple interest rate of 8% per year.

a. How much interest will she owe after 1 year? ---

b. How much interest will she pay in total, if the loan is for a period of 3 years?

---

7. Juan borrowed $1000 at a simple interest rate of 6% per year, and paid the interest monthly.
How much interest did he pay every month?

---

8. Mullica borrowed $2500 at a simple interest rate of 5% per year. How much interest did she owe if the loan was for a period of 8 months?

Name _____ Class _____ Date _____

Lesson 4.1.1

Ratios

California Standard: Number Sense 1.2

Example

a. What is the ratio of the shaded circles to the unshaded circles?
b. What is the ratio of the shaded circles to all the circles?
c. What is the ratio of the unshaded circles to the shaded circles?
d. What is the ratio of the unshaded circles to all the circles?
e. What is the ratio of all circles to the unshaded circles?

Notice that there are 3 ways to write ratios. The fraction form is the most used.

Solution

a. $\frac{3}{4}$ There are 3 shaded circles and 4 unshaded circles, so the ratio is 3 to 4 or 3:4 or $\frac{3}{4}$.

b. $\frac{3}{7}$ There are 3 shaded circles and 7 circles in total, so the ratio is 3 to 7 or 3:7 or $\frac{3}{7}$.

c. $\frac{4}{3}$ There are 4 unshaded circles and 3 shaded circles, so the ratio is 4 to 3 or 4:3 or $\frac{4}{3}$.

d. $\frac{4}{7}$ There are 4 unshaded circles and 7 circles in total, so the ratio is 4 to 7 or 4:7 or $\frac{4}{7}$.

e. $\frac{7}{4}$ There are 7 circles in total and 4 unshaded circles, so the ratio is 7 to 4 or 7:4 or $\frac{7}{4}$.

1. 

a. What is the ratio of shaded squares to unshaded squares? Write the ratio in 3 ways.

---

b. What is the ratio of unshaded squares to the total number of squares? Write the ratio in 3 ways.

---

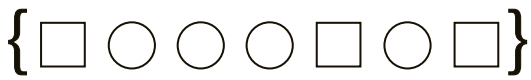
2. There are two trees in the yard. One is 5ft tall and the other is 8ft tall.
a. What is the ratio of the height of the shorter tree to the height of the taller tree? Write the ratio in three ways.

---

b. What is the ratio of the height of the taller tree to the height of the shorter tree? Write the ratio in three ways.

---

3. Write the answer to each of the following in three ways.



a. What is the ratio of the number of squares to the total number of objects in the set above?

---

b. What is the ratio of the number of circles to the total number of objects in the set above?

---

c. What is the ratio of the number of squares to the number of circles?

---

d. What is the ratio of the number of circles to the number of squares?

---

4. Joe is 5 ft tall. As he is walking, he notices that his shadow is 3 ft long. What is the ratio of the length of Joe's shadow to his height? Write your answer in three ways.

---

5. Jane's father is building her a doll house. It will be 4 ft tall, while their home is 13 ft tall. What is the ratio of the height of the doll house to the height of their home? Write your ratio in three ways.

---

6. On a golf course, the length of the first fairway is 187 yd and the length of the second fairway is 337 yd. What is the ratio of the length of the second fairway to the length of the first fairway?

---

7. A school house, which is 35 ft tall, casts a shadow that is 15 ft long. What is the ratio of the length of the shadow to the height of the school house?
Leave your answer in its simplest form. Write your answer in three forms.

---

8. The ages of the members of a Karate Club are displayed on the graph below.
Give all answers in their simplest form.

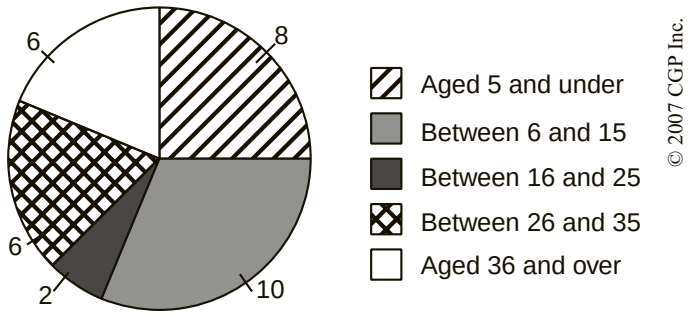
a. What is the ratio of members who are 5 or under, to members between 6 and 15?

---

b. What is the ratio of members aged between 26 and 35 to members aged 36 and over?

---

c. What is the ratio of members who are aged 36 and over to members aged 5 and under?



Name _____ Class _____ Date _____

Lesson 4.1.2

Equivalent Ratios

California Standard: Number Sense 1.2

Example

The sun, shining on the school building, makes a shadow 20 ft in length. The height of the school building making the shadow is 50 ft. What is the ratio of the length of the shadow to the height of the school?

Solution

The question asks for the ratio of the length of shadow of the building (20 feet) to the height of the building (50 feet). So this can be written as a ratio of 20 to 50 or $\frac{20}{50}$.

But this ratio has common factors and can be canceled down. The GCD of the two numbers in this case is 10, so divide each number in the ratio by this common factor, to reduce the fraction to its simplest form of $\frac{2}{5}$.

Therefore the final answer is **2 to 5, 2:5** or $\mathbf{\frac{2}{5}}$

Ratios should usually be left in their simplest form.

1. Simplify the following ratios:

a. 4 to 6 **b.** $\frac{8}{12}$ **c.** 12:20 **d.** 15 to 25 **e.** $\frac{100}{300}$ **f.** 75:100

---

2. Julio's age is 5, and his older brother is 15 years old.
What is the simplest form of the ratio of Julio's age to his brother's age?

---

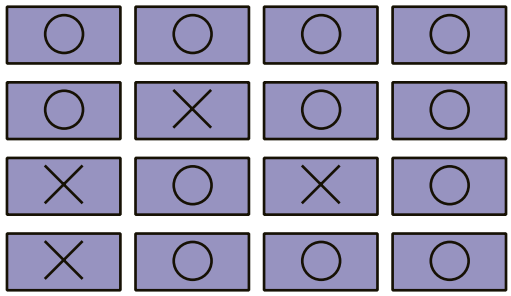
3. Of the 30 students in Mr. Frank's class, 18 are girls.
What is the simplest form of the ratio of girls to boys?

---

4. In Joe's marble collection, he has 12 red marbles, 8 green marbles, 16 blue marbles, and 24 multicolored marbles.
What is the simplest form of the ratio of the multicolored marbles to all the other marbles?

---

5. What is the ratio of cards with an X to cards with an O in the following picture?
Give your answer in its simplest form.



---

6. What is a number x such that the ratios $\frac{2}{3}$ and $\frac{x}{12}$ can be written as a proportion?

---

7. What is the number n such that the ratios 5:7 and n:21 can be written as a proportion?

---

8. What is the value of x such that the ratios $\frac{3}{7}$ and $\frac{18}{x}$ can be written as a proportion?

---

9. The ice on a glacier is melting at the rate of 2 in. every 3 days.
At that rate, how long will it take the 20 in. layer of ice to melt?

---

10. Rodney, who is 5 ft tall, casts a shadow of 2.5 ft. At exactly the same time that day, his home casts a shadow of 10 ft. How high is his home?

---

11. In a middle school class, there are 360 students, with a boy-to-girl ratio of 4 to 5.
Let g represent the number of girls. How many boys and how many girls are in the class?

Name _____ Class _____ Date _____

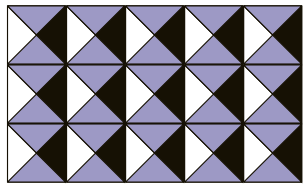
Lesson 4.1.3

Proportions

California Standard: Number Sense 1.3

Example

The pattern on the right is made up of tiles like the one shown.



a. Write a ratio showing the number of white triangles and blue triangles on the tile.

b. Write a ratio showing the number of white triangles and blue triangles in the whole pattern.

c. Use your two ratios to write a proportion.

Solution

a. There is 1 white triangle and 2 blue triangles.
So the ratio of white triangles to blue triangles on the single tile is **1 to 2**, or **1 : 2**, or $\frac{1}{2}$.

b. The whole pattern is made up of 15 tiles, so there are 15 white triangles and 30 blue triangles.
So the ratio of white triangles to blue triangles in the whole pattern is **15 to 30**, or **15 : 30**, or $\frac{15}{30}$.

c. The two ratios must be equal, so you can write a proportion: $\frac{1}{2} = \frac{15}{30}$

1. The number of girls and boys in one row at singing practice is shown below.

There are a total of five rows, all with the same number of girls and boys as the row above.

a. What is the ratio of girls to boys on one row? _____

b. What is the ratio of the total number of girls to the total number of boys? _____

c. Write a proportion to show the relationship between these ratios. _____

2. Alison poured 3 cups of coffee and 5 cups of tea at one table, as shown in the picture.

There were a total of 21 cups of coffee poured and 35 cups of tea. Is the overall ratio of cups of coffee to cups of tea in proportion with the numbers poured at the first table? Why?

_____

_____

3. Can the ratios $\frac{5}{6}$ and $\frac{66}{78}$ be written as a proportion? Explain your answer.

_____

4. The ratios 5:8 and x:56 are equivalent. Write a proportion involving x.

---

5. Determine the value of z if $\frac{z}{5} = \frac{10}{25}$. Show all your work.

---

6. Given $\frac{1}{3} = \frac{12}{y}$, determine y. Show all your work.

---

7. Rafael said that the ratio of boys to girls trying out for the cross-country team is $\frac{7}{8}$.
The team has 48 boys and 56 girls trying out. Was Rafael correct? Explain your answer.

---

8. The principal said that the ratio of girls to boys in the building was 180:195.
Jean said, "That is the same as 12:13." Was Jean correct? Explain your answer.

---

9. Tammy said that 2 cups of her pancake batter will make 14 pancakes. Tammy made x cups of batter that produced 42 pancakes. Write a proportion involving x.

---

10. Assume you burn about 300 calories when you run for 30 minutes.
How many calories will you burn if you run for 24 minutes? Explain your answer.

Name _____ Class _____ Date _____

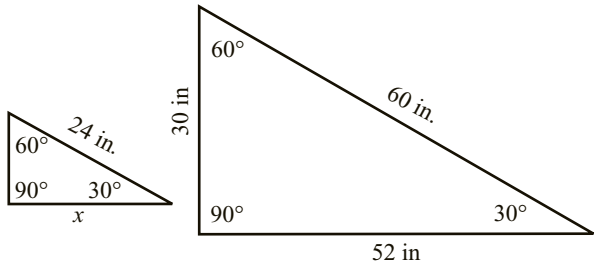
Lesson 4.1.4

Proportions and Cross-Multiplication

California Standard: Number Sense 1.3

Example

When two triangles have the same angle measures, then the two triangles are similar. When triangles are similar, corresponding sides are proportional. Determine x, the length of one side of the smaller triangle. Show your work.



Solution

The proportion is $\frac{x}{24} = \frac{52}{60}$

Remove fractions using cross-multiplication: $60x = 24 \times 52$

$60x = 1248$

Divide both sides by 60 to isolate x: $x = 1248 \div 60$

$x = 20.8$

Using the cross-multiplication method helps to solve problems involving proportions.

1. Use cross-multiplication to check if the following ratios are equivalent.

a. $\frac{5}{9}$ and $\frac{30}{56}$ ---------- **b.** $\frac{4}{7}$ and $\frac{20}{35}$ ---------- **c.** $\frac{3}{8}$ and $\frac{9}{21}$ ----------

---------- ---------- ----------

---------- ---------- ----------

2. Use cross-multiplication to find the value of the variable in each of the following proportions.

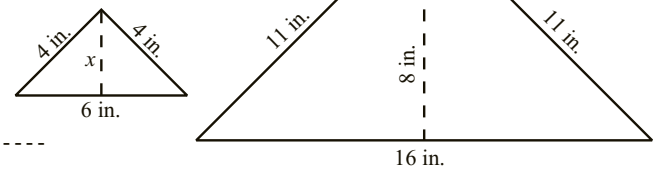
a. $\frac{y}{36} = \frac{42}{54}$ ---------- **b.** $4{:}12 = 11{:}z$ ----------

c. 12 to 20 = v to 35 ---------- **d.** $\frac{30}{u} = \frac{48}{88}$ ----------

3. Find the value of the variable in each of the following proportions.

a. $\frac{5}{c} = \frac{30}{36}$ ---------- **b.** $\frac{n}{8} = \frac{56}{64}$ ---------- **c.** $\frac{9}{8} = \frac{72}{p}$ ----------

4. These two triangles are similar. Use proportions to determine x, the height of the smaller triangle. Round your answer to the nearest whole number.



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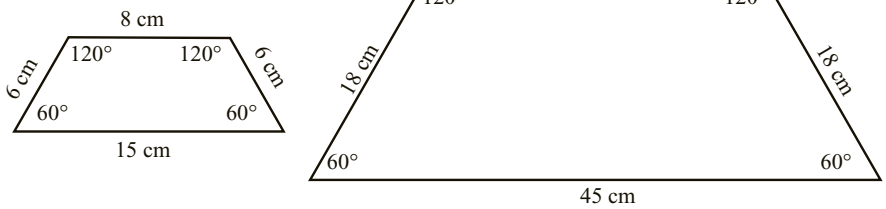
5. In these trapezoids, each ratio "smaller trapezoid side length : larger trapezoid side length" is the same. Determine x.



6. The ratio of the width to the length of a rectangle is 3 to 8. The length is 72 in. What is the width?

7. Jake noticed that 4 of the tulips in the first row of tulips were yellow and 5 were pink. If this ratio held for all of the tulips, how many yellow tulips would there be if there were a total of 100 pink?

8. The ratio of boys to girls in a classroom is $\frac{3}{4}$.

a. What is the ratio of girls to boys?

b. What is your answer when you multiply these two ratios together?

9. You are given the equation: $\frac{2}{3} = \frac{x}{9}$.

Least common multiples (LCMs) are covered in Lesson 3.4.5.

a. Find x by first multiplying both sides of the equation by the LCM of the fractions' denominators.

b. Go back to the original proportion and cross-multiply. What is the value of x using this method?

c. Is there any difference in working the problem by multiplying by the LCM or by cross-multiplication?

10. Francisco is placing marbles into gift bags. The ratio of yellow : green : white : red marbles in every bag is 2 : 1 : 8 : 6. All the marbles were used. If there were 42 red marbles in total, how many yellow, green, and white marbles were there?

Name _____ Class _____ Date _____

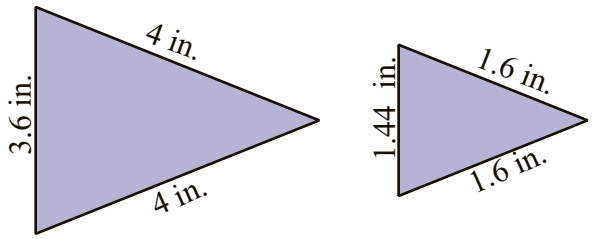
Lesson 4.2.1

Similarity

California Standard: Number Sense 1.3

Example

Are these 2 shapes similar?
Explain your answer.



Solution

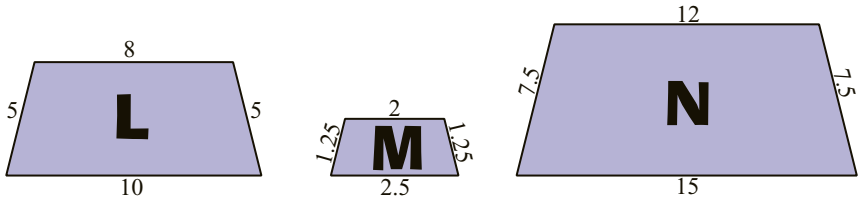
Work out the ratios of the lengths of the corresponding sides. If the shapes are similar, the ratios will be equal.

$\frac{4}{1.6} = 2.5$ and $\frac{3.6}{1.44} = 2.5$

The ratios of the lengths of the corresponding sides are equal.
Therefore **the shapes are similar**.

Note that you can also work out the ratio of two sides in the same shape and compare these — for example 3.6 to 4 is equivalent to 1.44 to 1.6

1. The three shapes below are all similar.



Find the ratios of the lengths of the corresponding sides between Figure L and:

a. Figure M _____

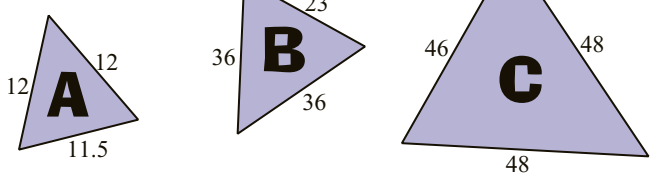
b. Figure N _____

2. Which of these three shapes are similar? Explain your answer.

_____

_____

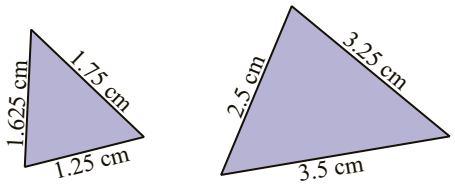
_____



3. Is the statement, "All squares are similar," true or false? Explain your answer.

---

4. Gus said the shapes below could not be similar because you have to rotate the 2nd one to put it into the same orientation as the first. Is Gus' statement correct? Explain why or why not.



---

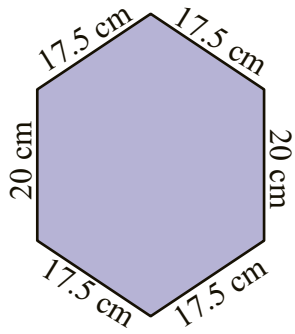
5. Travis had two similar triangles. The lengths of the sides on the first triangle were 9 inches, 7 inches, and 12 inches. The lengths of two of the sides on the second triangle were 22.5 inches and 17.5 inches. What must be the length of the third side on the triangle?

---

6. Halle drew a scale model of a hexagonal building.

a. What is the ratio of the length of a shorter side to the length of a longer side in this drawing?

The hexagon drawn here and the hexagonal building are similar. The length of the longer sides of the building is 12 meters.



b. What is the length of the shorter sides of the actual building?

---

c. What must the perimeter of the actual building be?

---

d. By how many meters would the perimeter change if the longer sides of the building were 16 meters, instead of 12 meters (but with the building remaining mathematically similar to the sketch)? Explain your answer.

Name _____ Class _____ Date _____

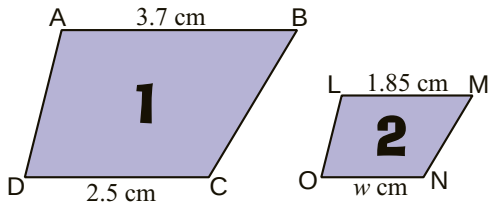
Lesson 4.2.2

Proportions and Similarity

California Standard: Number Sense 1.3

Example

Look at these similar shapes.
Write a proportion using the ratios AB:DC and LM:ON.
Solve your proportion to find w.



Solution

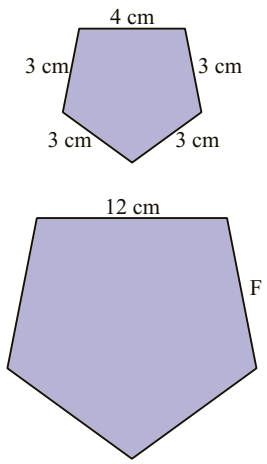
Since the shapes are similar, the ratio $\frac{3.7}{2.5}$ must equal the ratio $\frac{1.85}{w}$.

This means you can write a proportion: $\frac{3.7}{2.5} = \frac{1.85}{w}$.

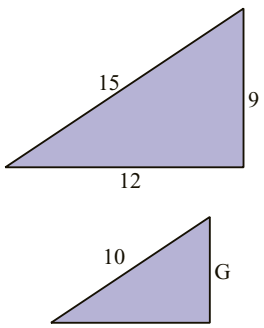
Now cross-multiply: $3.7w = 1.85 \times 2.5$, which means $w = \frac{1.85 \times 2.5}{3.7} = \mathbf{1.25\ cm}$

1. Find the length of the missing side in each of these pairs of similar shapes.

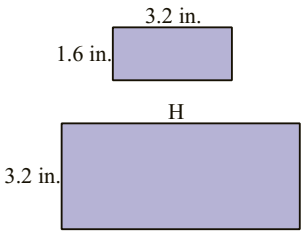
a. Side F



b. Side G

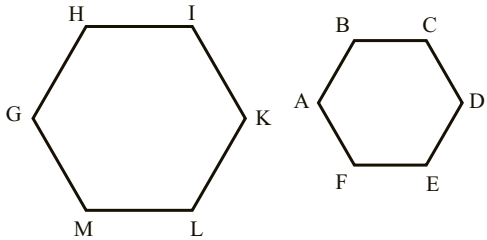


c. Side H



2. Figures ABCDEF and GHIKLM are similar, regular hexagons. The ratio of the lengths of the sides of ABCDEF to GHIKLM is $\frac{2}{3}$. If the side AB is 6 cm long, what is the perimeter of GHIKLM?

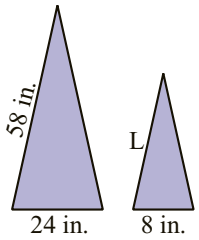
Remember that "regular" means all sides of a figure are the same length.



3. Caldwell is trying to determine the lengths of missing sides on two similar triangles. He first found that the ratio of the lengths of the two shortest sides was 4 : 1. If the lengths of the other two sides of the larger triangle are 8 cm and 14 cm, how long are the corresponding sides of the smaller triangle?

---

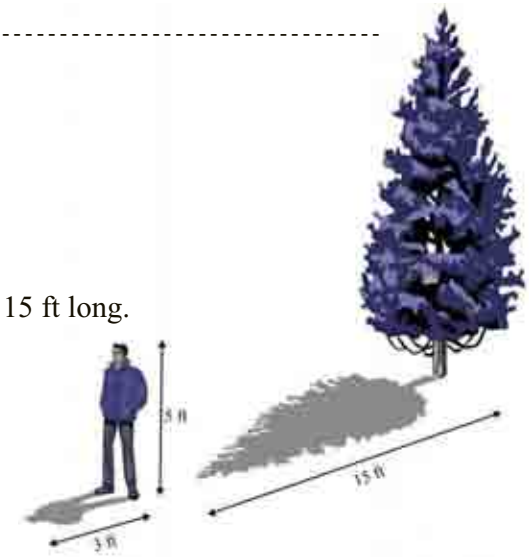
4. Kelly set up the proportion $\frac{58}{L} = \frac{8}{24}$ to find the length, L, of a side of the smaller triangle.



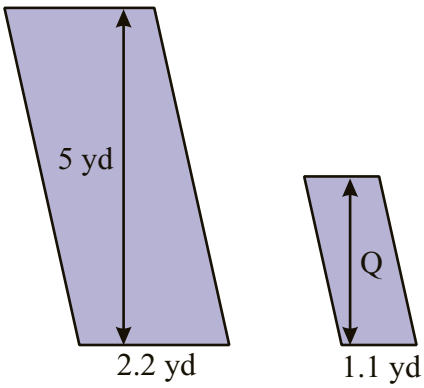
What did Kelly do wrong in setting up the proportion?

---

5. Maleko, who is 5 ft tall, casts a shadow 3 ft long.
He then notes that the shadow of the tree he is next to is 15 ft long.
How tall is the tree?



6. Brenda is moving from a condo with a small yard to a condo with an even smaller yard.
The diagram below shows the sizes of both yards, which are similar parallelograms.



You can find the length Q in exactly the same way as you'd find a side length.

a. What is the area of the first yard?

---

b. What is the length of side Q?

---

c. What is the area of the second yard?

---

d. What is the difference between the areas of the 2 yards?

Name _____ Class _____ Date _____

Lesson 4.2.3

Scale Drawings

California Standards: Number Sense 1.2, 1.3

Example

Mr. Franks is building a barn to hold his horses.
In a scale drawing, the barn measures 8 in. by 5 in., where the scale is 1 in. to 4 ft.
What are the real dimensions of the barn?

Solution

Call the length of the building l, and the width w. A proportion can be used to express each one:

$$\frac{1 \text{ in.}}{8 \text{ in.}} = \frac{4 \text{ ft}}{l \text{ ft}} \qquad \frac{1 \text{ in.}}{5 \text{ in.}} = \frac{4 \text{ ft}}{w \text{ ft}}$$

$$l = 4 \times 8 \qquad w = 4 \times 5$$

$$\boldsymbol{l = 32 \text{ ft}} \qquad \boldsymbol{w = 20 \text{ ft}}$$

Every inch of drawing converts to 4 ft of barn, so 5 in. of drawing converts to 4 × 5 = 20 ft of barn. The same reasoning can be applied for the length.

Therefore the dimensions of the barn are 32 feet by 20 feet.

1. On Briona's road map, the distance from Sacramento to San Diego is about 17 in.
The scale of the map says 1 in. to 30 miles.

a. About how far is it from Sacramento to San Diego, according to the map?

---

b. Another road map shows 11 inches from Redding, California, to San Francisco — a distance of about 220 miles. What is the scale on the map (how many miles does each inch represent)?

---

c. On the map in question b., the distance from Bakersfield to Los Angeles is 5.5 inches.
Use your answer to b. to work out how many miles Bakersfield is from Los Angeles?

---

2. Corbin has a drawing of a fish made with a scale of 1 cm to 14 cm.

a. The length of the fish in the drawing is 8 cm. What is the length of the actual fish?

---

b. The height of the fish in the drawing is 4.3 cm. What is the height of the actual fish?

---

3. Joe has a model of the Titanic which has a scale of 1 in. to 50 ft. His model is 17.6 in. long. About how long was the Titanic?

---

4. A map has a scale of 1 in. to 50 mi. The distance from Redding, California, to Bishop, California, on the map is 6.25 in. What is the real distance from Redding to Bishop?

---

5. Devin has a tree house drawing with a scale of 1 cm to 200 cm.

a. The width of the tree house in the drawing is 2.75 cm. What is the width of the actual tree house?

---

b. The height of the tree house in the drawing is 1.8 cm. What is the height of the actual tree house?

---

6. Mr. Wilson drew a 1 in. to 10 ft scale drawing of his goat pen. The drawing is 6 in. by 9 in. What is the perimeter of the goat pen?

---

7. The scale is missing on Jan's map and she needs to determine the distance from Sacramento to Eureka, CA. She knows the distance from Sacramento to Reno is about 200 miles, and it measures 5 in. on her map. The distance from Sacramento to Eureka measures about 7.5 in. on her map.

a. What is the scale of the map?

---

b. What is the approximate distance from Sacramento to Eureka?

---

8. Mr. Jones is going to raise rabbits on his farm to sell to grocery stores. He decides he needs a pen for the rabbits with an area of 700 m^2. One side of the rectangular pen is going to be 20 m long.

a. What will the perimeter of the pen be?

---

b. Mr. Jones makes a scale drawing of the pen, with a scale of 1 cm to 5 m. What will the dimensions of this drawing be?

Name _____ Class _____ Date _____

Lesson 4.3.1

Customary and Metric Units

California Standards: Algebra and Functions 2.1, Mathematical Reasoning 3.1

Example

A mural is 2 yards tall. What is the height of the mural in:

a. feet?

b. inches?

Solution

Use the conversion chart opposite.

a. You know that 1 yard is equal to 3 feet, so 2 yards would be 2×3 which is **6 feet**.

b. You know that 1 foot is 12 inches, so 6 feet would be 6×12 which is **72 inches**.

Customary Lengths

1 mile = 1760 yards
1 yard = 3 feet
1 foot = 12 inches

1. Terrence said that when you convert a quantity from smaller units to bigger units, you multiply the number by a conversion factor greater than 1.
Is Terrence's statement correct? Explain why or why not.

---

2. Dividing by $\frac{1}{100}$ is the same as multiplying by what number?

---

3. Convert the following measurements using the conversion charts.

a. 6 miles to yards

---

b. 5 yards to inches

---

4. Rosalyn said, "My cat from end of tail to nose is 620 mm long."
How many cm long is the cat?

Metric Lengths

1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

5. While driving in Europe, Mr. Roberts sees a sign: **Berlin 100 km**

How many meters is he from Berlin?

---

6. A sofa has a length of 213 centimeters.
Use the conversion chart to convert this length to:

a. meters. Show your work.

---

b. millimeters. Show your work.

---

7. A truck had 312 centimeters of space left for the sofa in Exercise 6, and plenty of width and height. What is the length the truck had left after the sofa was placed onto it? Write your answer in:

a. meters

---

b. millimeters

---

8. In five games, the halfback has run for 234 yd, 199 yd, 218 yd, 302 yd, and 188 yd.
How many more feet does he have to run to have run one mile?

---

9. Arnoldo is buying snacks for a party. He finds crackers in 3 lb boxes for $2.37, and in 5 lb boxes for $3.75. Which is the better buy?

Name _____ Class _____ Date _____

Lesson 4.3.2

Conversions and Proportions

California Standard: Algebra and Functions 2.1

Example

When you are in a car going 60 mi/h, you are going 88 ft/s. 88 ft is how many yards?

Solution

Since 3 ft = 1 yd, the proportion to be solved is: $\frac{3}{1} = \frac{88}{x}$

$3x = 88$

$x = 88 \div 3 = 29.\overline{3}$

So, 88 ft is $\mathbf{29\frac{1}{3}}$ **yd**.

There are some useful conversion factors in Lesson 4.3.1.

1. A team's fullback ran for 153 yd in last night's football game. How many feet did he run?

---

2. A road sign says, **Construction ¼ mi** How many feet before the construction?

---

3. A rogue wave at sea is measured as 28 m high. How many cm high is this wave?

---

4. Victoria Falls in Africa is about 1.7 km wide. About how many meters wide is the waterfall?

---

5. While driving in France, Mr. Ostegard sees a sign which has a left turn arrow and says, "400 m."

a. How many km away is the left turn? _____

b. How many cm away is the turn? _____

6. When a car is traveling at 60 miles per hour it takes about 300 ft to stop. How many yards does it take to stop?

---

7. It takes an extra 84 feet to stop from 70 mi/h, instead of 60 mi/h. How many extra yards is this?

If ft is in the numerator of the first ratio, then ft must be in the numerator of the second ratio.

8. The peak of Mt Wilson, California, is 5710 ft above sea level.

a. How many yards high is this peak? ---

b. How many miles high is this peak? ---

c. How many inches high is this peak? ---

9. A luxury yacht is 164 ft long.

a. How many yards is this? ---

b. How many inches is this? ---

c. How many miles is this? ---

Example

Mt Shasta, the fifth highest peak in California, is 4322 meters high.
How many centimeters high is Mt Shasta?

Solution

The units involved are cm and m, which are linked by the relationship: 1 m = 100 cm

Therefore the following proportion can be used: $\frac{1}{100} = \frac{4322}{x}$

$x = 4322 \times 100$

$x =$ **432,200 cm**

10. Mt Lassen, the last California mountain to erupt (1914-1921) is 3189 m high.

a. How many centimeters high is Mt Lassen? ---

For a decimeters reminder, see Lesson 4.3.1

b. How many decimeters high is Mt Lassen? ---

c. How many millimeters high is Mt Lassen? ---

11. A standard athletics track is 400 m long.

a. How long is this in cm? ---

b. How long is this in mm? ---

c. How long is this in km? ---

12. Insert one of these three symbols (< , = , >)between each pair of numbers.

a. 24 ft _____ 289 in. **b.** 360 yd _____ 1080 ft **c.** 560 in. _____ 47 ft

d. 31,680 ft _____ 6 mi **e.** 68,650 ft _____ 13 mi **f.** 107 yd _____ 322 ft

Name _____ Class _____ Date _____

Lesson 4.3.3

Converting Between Unit Systems

California Standard: Algebra and Functions 2.1

Example

Mt. St Helens was 9677 ft high before the explosive eruption of May 18, 1980. After the eruption it was 8364 ft high. How many meters were blown off the top of Mt. St Helens?

Solution

9677 – 8364 = 1313 feet

Calculate the feet removed by subtracting the final height from its initial height.

Using the conversion 1 m = 3.28 ft, the proportion becomes: $\frac{1}{3.28} = \frac{x}{1313}$

$x = \frac{1313}{3.28}$ **Isolate the variable**

x = **400.3... meters**

Notice in the proportion that both numerators are in meters and both denominators in feet.

Conversion Chart

1 meter	= 1.09 yards
1 yard	= 0.91 meters
1 meter	= 100 cm
1 mile	= 1.61 km
1 km	= 1000 meters
1 meter	= 1000 mm
1 cm	= 0.39 in.
1 mm	= $\frac{1}{1000}$ m

Use the conversion chart to help you answer questions 1 to 4.

1. The driveway at Timmy's house is 8.5 meters long.
Convert this distance to yards.

---

2. Camilla uses 15 yards of fabric to make new curtains for her home.
Convert this length to centimeters.

---

3. The distance from Kyle's house to his aunt's house down the road is 0.85 miles.
Convert this distance to millimeters.

---

4. Janice made a giant straw that was 5280 millimeters long.

a. Convert this length to inches.

---

b. Convert this length to yards.

---

5. Determine the height of each of the following mountains in meters.

a. Mt Rainier — 14,410 ft

---

b. Mt Wilson — 5710 ft

---

c. Mt Shasta — 14179 ft

---

d. Mt Hood — 11,249 ft

---

e. Mt Adams — 12,278 ft

---

f. Mt Lassen — 10, 457 ft

---

6. On August 7th 1995, a world athletics record for triple jump was set in Gothenburg. The distance was 18.29 m. How far is this in feet?

---

7. Francis measures the distance between two cities on a map as 5 in. However, the scale is in cm. How many cm is 5 in.?

---

8. Convert 50 mi/h to km/h.

In this case the "per hour" can be ignored as it does not change between the units.

9. Write 60 miles per hour in miles per second.

Remember: 60 minutes = 1 hour
60 seconds = 1 minute

---

10. The Rodriguez family thought they would drive from San Diego, California, to Mexico City, Mexico, to visit relatives. They checked the map, and found that the distance from Tijuana, Mexico, (just across the border from San Diego) to Mexico City was 2836 km.

a. How many miles is it from Tijuana to Mexico City?

---

b. If they can average 55 mi/h, about how many hours will it take them to make the trip?

---

11. The plans for a new house were drawn using a scale of 1 in. to 10 ft. On the plans, the house measures 6 in. by 8 in. When the house was actually built, the builder assumed the scale was 1 in. to 10 m, and built the house with that scale. What were the dimensions in feet of the house when it was built?

Name _____ Class _____ Date _____

Lesson 4.3.4

Other Conversions

California Standard: Algebra and Functions 2.1

Example

Convert 123 °F to °C.

Solution

The formula used to convert from Fahrenheit to Celsius is $C = (F - 32) \div 1.8$

Substitute 123 as the value for F.

$C = (123 - 32) \div 1.8$
$C = (91) \div 1.8$
$C = 50.56$
So, 123 °F is equal to **50.56 °C**

1. Explain how to type in the number –49.5 on your calculator.

---

2. Convert each of these temperatures to degrees Celsius.
If necessary, round answers to the nearest hundredth.

a. 41 °F

---

b. 293 °F

---

c. 0 °F

---

d. 212 °F

---

e. 99.5 °F

---

f. 201.2 °F

---

3. Convert each of these temperatures to degrees Fahrenheit.
Round each answer to the nearest hundredth.

a. –27 °C

---

b. 110 °C

---

c. 0 °C

---

d. 82 °C

---

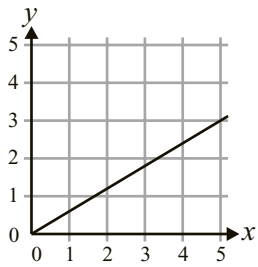
e. 27 °C

---

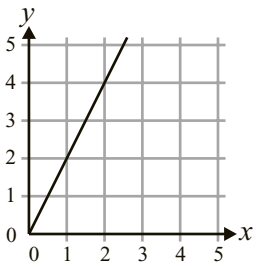
f. –4 °C

---

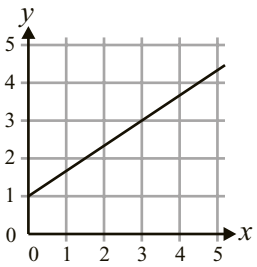
4. Use the diagrams below to answer the question.



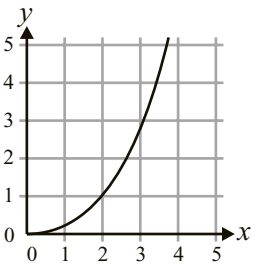
Proportional



Proportional



Not Proportional

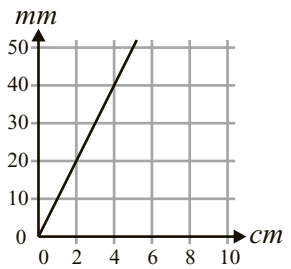


Not Proportional

Using the above graphs and your knowledge of proportionality, write down the two primary features of proportional graphs.

---

5. Explain how to use a graph to make a conversion.
If it helps with your explanation, refer to the graph shown here.



Kendall uses the formula $y = 1.4x + 2.80$ to determine y, the price ($) she should charge for each skateboard at her store. x represents the amount that Kendall paid for the skateboard.

6. What price should Kendall charge for a skateboard that cost her $28.54?

---

7. Kendall used the formula to price a different skateboard at $44.80.
What was the cost of the skateboard to Kendall?

---

8. By how much would your answer to the last question change if Kendall used the formula $y = x + 2.80$ instead?

Name _____ Class _____ Date _____

Lesson 4.4.1

Rates

California Standard: Algebra and Functions 2.2

Example

In the Seniors races, Mr. Brisco ran the 100 yd race in 25 s.
What was his speed (or rate of movement) in yards per second?

Solution

If he goes 100 yards in 25 s, his rate can be written as:

$$\frac{100\text{ yd}}{25\text{ s}} = \frac{100\text{ yd} \div 25}{25\text{ s} \div 25} = \frac{4\text{ yd}}{1\text{ s}}$$

Divide through by the greatest common divisor.

So his speed (or rate) is **4 yd/s.**

"4 yd/s" is read as "four yards per second."

Or how many books per hour did they sell?

1. The book store sold 40 books in 8 hours. At what rate did the book store sell books?

---

2. James went to the store and bought a 5 lb roast for $17.25. How much did James spend per pound?

---

3. Preston put 12 gallons of gas in his car, and it cost him $36.72.
How much did Preston pay per gallon?

---

4. In a school, there are 658 students and 28 teachers. How many students per teacher are there?

---

5. John drove 375 miles in 7.5 hours. What was his rate of movement per hour?

---

6. Jimmy goes to the store to buy pretzels for a party. He can buy 2 lb packages for $2.72 or 5 lb packages for $7.10. Which package is a better buy, and why?

---

7. Elias practiced the clarinet for 260 minutes over 2 weeks.
His sister practiced the piano for 360 minutes over 3 weeks.
On average, who practiced for longer each week?

---

8. At a driving range, Carrie can pay either $8 for a large bucket with 94 balls or $6 for a small bucket with 54 balls. How many balls does Carrie get for each dollar, if she buys the:

a. large bucket?

---

b. small bucket?

---

9. Duane wants to know the rate at which he stains boards on a fence. If he stains 48 boards in 3 hours, what is the rate at which Duane is working, measured in:

a. boards per hour?

---

b. boards per minute?

---

c. minutes per board?

---

10. A store claims that for every 160 female customers, they get 35 male customers.

a. Find the number of females that come in for every one male.

---

b. Find the number of males that come in for every one female.

---

c. The number of female customers who entered the store in 2 hours was 576.
Using this figure, estimate the rate of male customers per hour?

Name _____ Class _____ Date _____

Lesson 4.4.2

Using Rates

California Standards: Algebra and Functions 2.2, 2.3

Example

Avril goes to the grocery story to buy fruit. Bananas are listed as costing \$0.89 per pound. Avril buys 5 pounds of bananas. How much does Avril spend on the bananas?

Solution

The rate means \$0.89 per pound means: $\frac{\text{number of dollars}}{\text{number of pounds}} = 0.89$.

So to find the total number of dollars, multiply \$0.89 per pound by 5.

\$0.89 × 5 = \$4.45 in total.

Therefore the total cost of the bananas is **\$4.45**.

1. Ronan is buying fabric. Each yard will cost him \$1.99. What does Ronan need to know to determine the total amount of money he will spend on the fabric?

---

2. Jackie's basketball team scored 78 points during their first game of the year. If they continue to score points at this rate, how many will they have scored after $\frac{1}{2}$ of a 24-game season?

---

3. Kelly is purchasing kites for her class. She needs a total of 28 kites. The kites come in packs of 2 for \$18.00. What steps does Kelly need to take to determine

a. the cost of 1 kite?

---

b. the cost of 28 kites?

---

4. A mailroom clerk delivered 7 packets of papers to each conference room in a building. If there are a total of 15 conference rooms, how many packets did the clerk deliver?

---

5. A windmill completed 16 rotations each minute for a total of 20 minutes.
What is the total number of rotations the windmill completed?

---

6. A wedding planner said that each meal at the wedding would cost $15.95.
If a total of $1754.50 was spent on the meals, how many people could be fed at the wedding?

---

7. A mixer spun 150 times per minute.
If it spun a total of 2100 times, for how many minutes did it spin?

---

8. Terri says that you can calculate the total number of times a person's heart beats in x minutes by dividing the heart rate (measured in beats per minute) by x.
Is Terri correct? Explain your answer.

---

9. Drew buys 6 bags of vegetables at the supermarket each week.
Each bag normally costs $2.50, but this week they are on a "buy two get one free" offer.

a. Express the price of vegetable bags in "bags per dollar" for a normal week of shopping.

---

b. What rate of "bags per dollar" could Drew get this week?

---

c. How much money will Drew save on vegetables this week?

---

10. A motorcyclist traveled at a rate of 60 miles per hour for 3 hours, then took a break for lunch for 45 minutes. She then traveled for another amount of time at 60 miles per hour before reaching her destination. If she traveled a total of 270 miles, and left at 9:30 a.m., at what time did she arrive at her destination?

Name | Class | Date

Lesson 4.4.3

Finding Speed

California Standards: Algebra and Functions 2.2, 2.3

Example

A giraffe ran a distance of 0.1 miles in 18 seconds.
Find the giraffe's speed in mi/h.

Solution

You need to use the formula: $\textbf{Speed} = \dfrac{\textbf{Distance}}{\textbf{Time}}$.

To find the speed in mi/h, you need the distance in miles, and the time in hours.

Convert 18 seconds to hours by dividing by $60 \times 60 = 3600$.
This means the time in hours is $18 \div 3600 = 0.005$ hours.

Now you can work out the giraffe's speed: $\text{speed} = \dfrac{0.1 \text{ miles}}{0.005 \text{ hours}} = 20 \text{ mi/h}.$

1. Scientists tracking a mallard duck as it migrates South measure it traveling 2520 km during a nonstop flight of 28 hours. What was the average speed of the duck?

---

2. Amie is training for a bicycle race. She can go 30 miles in $1\frac{2}{3}$ hours.
If she travels at the same speed throughout this entire time, how fast will she be traveling?

---

3. Flying nonstop from Los Angeles to New York, a distance of about 3185 miles, takes about $6\frac{1}{2}$ hours.
What is the plane's average speed?

---

4. Mr. Cruise drove 288 miles in 8 hours. What was his average rate of travel in miles per hour?

---

5. Gwen rode her bicycle for 4 hours at the same speed. She stopped riding after she had ridden for 36 miles. What was Gwen's speed in miles per hour?

---

6. Mrs. Smith walks 2 miles every day and it takes her 40 min.
Assuming she walks at a constant speed, how fast does she travel in miles per hour?

---

7. Mr. Franco walks around a 400 meter track twice and it takes him 10 min.
What is his speed in kilometers per hour?

---

8. A helicopter, flying at a constant speed, travels 38 miles in 8.4 minutes.
What is the speed of the helicopter in miles per hour?

---

9. During a sprint, Giancarlo covers 25 ft in one second.
What is his speed in mph?

---

1 mile = 1760 yards
1 foot = 3 yards
60 seconds = 1 minute
60 minutes = 1 hour
Other conversions are listed in the textbook

10. A quarter horse can run at a speed of 47.5 miles per hour.

a. What is this speed in miles per minute?

---

b. What is this speed in miles per second?

---

c. Given there are 8 furlongs in a mile, and 14 days in a fortnight, what is the speed of the quarter horse in furlongs per fortnight?

Name | Class | Date

Lesson 4.4.4

Finding Time And Distance

California Standards: Algebra and Functions 2.2, 2.3

Example

Joellen hiked up a hill at a speed of 3 miles per hour and took 2.5 hours.
She walked down the same distance of the hill at a speed of 5 miles per hour.
How long did it take Joellen to walk down the hill?

Solution

To calculate the time taken to walk down the hill, distance and speed are required.
The speed is given, and the distance can be calculated from the information about hiking up the hill:
Using the formula: Distance = Speed × Time,

$$\text{Distance} = \frac{3 \text{ miles}}{\cancel{\text{hour}}} \times 2.5 \ \cancel{\text{hours}} = 7.5 \text{ miles}$$

So, to calculate the time taken to walk down the hill, use these figures
(5 miles per hour, and 7.5 miles) in the formula: $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{7.5 \text{ miles}}{5 \frac{\text{miles}}{\text{hour}}} = 1.5 \text{ hours}$$

So it took Joellen **1.5 hours** to walk down the hill.

1. Mr. Jones must go from Bakersfield to Palm Springs. He believes that he can average 50 mi/h, and that it will take him 4.5 hours. If he is correct, how far is it from Bakersfield to Palm Springs?

---

2. When Canadian Geese migrate South, they travel at about 60 mi/h. If it takes 6 days, with 5 hours flying per day, for some of the geese to reach their destination, how far do they fly?

---

3. Mr. Sanchez drives a large 22-wheeler freight truck. He can average about 55 mi/h and he usually drives about 400 mi a day. About how many hours per day does he drive?

---

4. An airplane traveling at 450 mi/h goes 7200 mi.
How long does it take the plane to cover this distance?

---

5. Flying from Sydney, Australia, to Los Angeles, California, takes about 15 hours, including a one-hour stop in Honolulu, Hawaii. The airplane averages about 475 mi/h when in flight.
How far is it from Sydney to Los Angeles?

---

6. A train left Town A traveling at 35 miles per hour.
How long would it take the train to travel 420 miles at this speed?

---

7. A boat tour lasted 40 minutes. The boat traveled at a constant rate of 3 miles/hour.
How far did the boat travel during the tour?

---

8. Franco said that when the speed and distance are known, you only have to multiply them to get the time. Is Franco's statement correct? Explain why or why not.

---

9. A frog took 3.5 minutes to cross the finish line at a frog-jumping contest. The frog jumped at a rate of 1 meter/25 seconds.

a. What was the distance of the jumping contest?

---

b. What was the frog's speed in centimeters per minute?

---

10. Trina and Jake are each running half of a marathon. Trina runs the first 13.05 miles at an average speed of 6 miles per hour. Jake runs the second 13.05 miles at an average speed of 7 miles per hour.
What is their combined time for the marathon?
Give your answer in hours and minutes, to the nearest minute.

Name | Class | Date

Lesson 4.4.5

Average Rates

California Standards: Algebra and Functions 2.2, 2.3

Example

Juwan ran the first half of a 10 mile race at 8.3 miles per hour and the second half of the race at 6 miles per hour. What was Juwan's average speed for the race?

Solution

For each half of the race, calculate the unknowns in the formula: Distance = Speed × Time

First half: Distance = 5 miles, Speed = 8.3 miles per hour, Time = $\frac{5}{8.3}$ = 0.602... hours

Second half: Distance = 5 miles, Speed = 6 miles per hour, Time = $\frac{5}{6} = 0.8\overline{3}$ hours

Now, the average speed can be worked out using the totals:

$$\text{Speed} = \frac{\text{total distance}}{\text{total time}} = \frac{5+5}{0.602..+0.8\overline{3}} = \frac{10}{1.436..} = 6.96...\frac{\text{miles}}{\text{hour}}$$

So, Juwan's average speed for the race was **6.97 miles per hour** (2 d.p.)

Work out the average speed for the entire journey in Exercises 1 to 4.

1. A canoeist paddled up a 0.75 mile stretch of a river at 1.5 miles per hour, then turned and paddled back down to where he started at 5 miles per hour.

2. An empty semi-truck traveled at an average speed of 65 mi/h for a 500 mile trip. After being loaded with cargo, the semi could only travel at an average speed of 60 mi/h, but it made a journey of 1000 miles.

3. A fish swims for 5 miles at a speed of 25 miles per hour, and then 10 miles at a speed of 15 miles per hour.

4. Angela drove home in a traffic jam. She traveled a quarter mile at a rate of 2 miles per hour, made it through the jam, and then traveled the remaining 4.5 miles at a rate of 50 miles per hour.

5. Arlene needs to determine the average speed of a journey that was split into 3 parts.
What steps should Arlene take if the speed and distance is known for each part?

---

6. Brennan knew that the rate of the first part of a journey was 6 miles per hour, and that the rate of the second part was 8 miles per hour. Explain why the average speed for the journey might not equal 7 miles per hour?

---

7. Alvin ran 8 laps around a $\frac{1}{4}$ mile track at a pace of 1 lap every 2 minutes.
Then he ran 5 more laps around the same track at a pace of 1 lap every 2.5 minutes.
What was Alvin's average speed over the 13 laps?

---

8. Gwen rode her snowmobile up a mountain trail at a speed of 20 miles per hour.
She rode it down the mountain trail at a speed of 30 miles per hour.
The mountain trail was 15.8 miles long.
What was Gwen's average speed for the trip?

---

9. By how much would Gwen's speed from Exercise 8 change if she rode down the hill at 35 miles per hour instead of at 30?

---

10. Mr. Anderson took a trip to visit his friend in Canada. His first flight was on a big jet. The flight lasted 2.75 hours at an average speed of 475 miles per hour. His second flight was on a smaller plane and lasted 1.5 hours at an average speed of 300 miles per hour. Mr. Anderson had a 1-hour break between flights. What was Mr. Anderson's average speed over the entire trip, not including the break?

Name _____ Class _____ Date _____

Lesson 5.1.1

Median and Mode

California Standard: Statistics, Data Analysis, and Probability 1.1

Example

11 students ran the hundred yard dash. Their times for the race, in seconds, are shown below. Find the median and the mode of the times run by the students.

{11.4, 12.7, 10.9, 13.8, 12.7, 15.4, 16.1, 12.7, 14.9, 17.2, 15.4}

Solution

Put the numbers in order from the smallest to the largest:

{10.9, 11.4, 12.7, 12.7, 12.7, 13.8, 14.9, 15.4, 15.4, 16.1, 17.2}

When you put a set of numbers in order, the median is the middle value.

There are 11 values in this data set, so the median is the 6th value, which is 13.8.

The mode is the value that occurs most often. In this data set the mode is 12.7, because it occurs three times — more than any other value in the set.

Median = 13.8 seconds, Mode = 12.7 seconds

1. Seven students had to take a makeup exam. Their scores were as follows:

{94, 71, 82, 68, 75, 79, 87}.

What is the median of this set of scores? _____

2. Janice made six calls on her cellphone. The duration of these calls, in minutes, is shown below.

{9, 4, 10, 2, 7, 5}. ← The median is not necessarily in the list of numbers.

Find the median duration of Janice's six calls. _____

3. A group of 8 students were asked to name their favorite color. They gave the following answers:

{yellow, red, green, red, blue, pink, green, red}

a. What is the mode of this data set? _____

b. Can you find the median of this data set? _____

4. Lemar asked each of his friends how far away from school they lived, in blocks. This is the list that he made: {4, 7, 2, 4, 8, 6, 2, 4, 5, 2, 6, 9}

Find the mode of Lemar's data set. _____

5. Find the median and the mode of the following data set.

{3, 6, 8, 10, 6, 1, 10, 6, 11, 7, 5, 6, 10} _____

6. Find the median and the mode of the following data set.

{6, 9, 12, 6, 9, 12, 6, 9, 12} _____

7. The shoe sizes of a group of children living on the same street are listed below.
{2, 2, 2, 2, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5}

a. What is the mode of this data set? ----------

b. What is the median shoe size of the group of children? ----------

8. The data set below shows the number of points Eduardo scored in each of the football games he played in during this semester.
{6, 2, 13, 5, 8, 13, 9, 11, 13, 0, 5, 1, 11, 3}

a. What was Eduardo's median game score? ----------

b. What is the mode of the data set? ----------

9. The data set below shows the ages of children checking out books at a library on a Saturday.
{4, 8, 8, 9}

a. Find the mode of the data set. ----------

b. Find the median age of children checking out books that Saturday. ----------

c. If a 6-year-old checked out a book, would the median change? ----------

d. If another 9-year-old checked out a book, would the mode change? ----------

Example

The median of the data set below is 7. What is the missing value?
{5, 2, 11, ___, 4, 13}

Solution

Put the numbers that you know in order from the smallest to the largest:
{2, 4, 5, 11, 13}
In a data set with six numbers, the median will lie midway between the third and fourth values. The third value at the moment is 5. Since 5 is two less than 7, to make it the median, the missing value must be two more than 7.

So the missing value is 9.

10. Jada asked eight people what their favorite fruit is. The answers she got are shown below.
{apple, pear, kiwi, peach, ___, kiwi, apple, strawberry}
The mode of Jada's data set was kiwi. What is the missing value? ----------

11. The median of the data set below is 12. What is the missing value?
{9, ___, 13, 15} ----------

12. The mode and median of the data set below have the same value. What is the missing number?
{4, 15, 7, 13, ___, 10}

Name _____ Class _____ Date _____

Lesson 5.1.2

Mean and Range

California Standard: Statistics, Data Analysis, and Probability 1.1

Example

Roger earned the following scores on a series of math tests: {72, 85, 97, 55, 64, 89}. What was his mean score for the six exams?

Finding the mean is often referred to as finding the average

Solution

72 + 85 + 97 + 55 + 64 + 89 = 462
462 ÷ 6 = 77

Add all the values in the data set
Then divide by the number of values in the data set

Roger's mean score for the six exams was 77.

1. Find the mean of the set of data shown below.
{15, 18, 22, 56, 41, 28}

2. Javier needs a grade average of 90% or more to receive an A grade for his math course. His test scores are {88%, 94%, 82%, 98%, 95%, 83%}. Will Javier get an A? Explain your answer.

3. Kerry works as a waitress on the weekend. The data set below shows how much she earned on five successive Saturdays, including tips. What were Kerry's mean Saturday earnings?
{$94, $87, $56, $91, $82, $100}

4. Francine needs to average 90% on the 5 tests in her math class. Her scores on the first four tests she took are 92%, 94%, 88%, and 85%. What is the minimum score she can get on the fifth test and still have an average of 90%?

5. Wesley measures the temperature outside his house on four days in Summer. The temperatures he records are 91 °F, 86 °F, 89 °F, and 88 °F. What was the mean temperature over the four days?

6. The mean of three numbers is 23. Two of the numbers are 24 and 19. What is the third number?

Example

Find the range of the following data set.

{4, 9, 3, 8, 1, 7, 3, 1}

Solution

The range of a data set is the difference between the greatest and least values in the set.
The range of this data set is 9 – 1 = **8**.

7. The range of a data set is 15. The smallest value in the set is 8. What is the largest value in the set?

---

8. Jenna is 15 years old and plays on a baseball team. The age range of the team is 3 years. If Jenna is the oldest player on the team, how old is the youngest player?

---

9. You are told that the range of a data set is 9. Can you tell from this how many numbers are in the set?

---

10. A group of math students measure their hand spans. The smallest hand span in the group is 14 cm. The range of all the hand spans in the group is 7 cm. What is the largest hand span in the group?

---

11. Mr. Gonzalez gave his class a test. The highest possible score on the test was 100 points. The range of points in the test was 60. Is it possible that someone scored a 25 on the test? Explain.

---

12. The data set below show the ages of the brothers and sisters of some children in a third grade class.

{3, 3, 4, 5, 5, 5, 6, 6, 8}

a. Find the range of the data set.

---

b. Find the mean of the data set.

---

c. Find the mode of the data set.

---

13. The data set below shows the number of points Charo scored in her first 7 basketball games this season:

{13, 11, 14, 9, 16, 4, 10}.

Find the mean and range of this data set.

---

14. This ordered data set has a range of 14, and a mean of 10: {3, 5, 6, 8, 9, x, 15, 16, y}.
What are the missing values, x and y?

Name _____ Class _____ Date _____

Lesson 5.1.3

Extreme Values

California Standards: Statistics, Data Analysis, and Probability 1.1, 1.3, 1.4

Example

On a series of five tests, Chin's scores were {0, 94, 88, 90, 88}. Chin felt it was unfair to include the score of 0, because he was in the hospital at the time, and could not take the test. Calculate Chin's mean test score with and without the 0 mark.

Solution

With 0 in the set: $(0 + 94 + 88 + 90 + 88) \div 5 = 360 \div 5 = \mathbf{72}$
With 0 taken out of the set: $(94 + 88 + 90 + 88) \div 4 = 360 \div 4 = \mathbf{90}$

Zero is clearly an outlier. Whether you include it or not greatly affects Chin's score.

1. In which of the following data sets might 100 be considered an outlier?

A. {14, 20, 11, 15, 100}
B. {88, 95, 100, 105, 101}
C. {564, 728, 637, 100, 598}
D. {100.1, 99.81, 101.4, 100, 100.52}

---

2. The data set below shows the number of people that were in six vehicles stopped at random for a survey.
{3, 4, 6, 2, 5, 46}

a. Which value in the data set is an outlier? _____

b. What is the mean with the outlier included? _____

c. What is the mean if the outlier is excluded? _____

3. Mr. Miller gives his class quizzes during the semester. Each quiz is worth 10 points. Kathleen's quiz scores are {1, 8, 9, 9, 10, 9, 10, 8}.

a. What is Kathleen's average quiz score? _____

b. Mr. Miller drops the lowest quiz score when he calculates the grade. What is Kathleen's new average quiz score? _____

4. The data set below shows the prices of eight houses sold in the same town during one month.
{$159,000; $174,000; $160,000; $743,000; $209,000; $138,000; $149,000; $199,000}

a. Find the median of the data set. _____

b. Find the mean price of the houses. _____

c. Explain whether the mean or the median is a better measure of central tendency to use for this data set.

---

5. The data set below shows the ages of a group of people waiting in line to pick up college ID cards.
{21, 61, 23, 19, 24, 20, 21}

a. Find the median age of the people in line.

b. Find the mean age of the people in line.

6. The missing value, a, in the data set below is an outlier. When a is excluded, the mean of the set is 49.
{45, 54, 53, 49, 44, a}

a. Give a possible value of a that will lower the mean if it is included.

b. Give a possible value of a that will raise the mean if it is included.

7. The data set below shows the prices of seven paintings sold at an art sale.
{\$98, \$106, \$104, \$105, \$107, \$98, \$110}

a. Find the mode of this data set.

b. Find the median price of the paintings.

c. Find the mean price of the paintings.

d. Explain why the mode is not a sensible measure of central tendency to use for this data set.

8. The missing value, x, in the data set below is an outlier. When x is excluded, the mean of the data set is 22. When x is included, the mean of the data set is 30. What is the value of x?
{15, 20, 22, 25, 28, x}

9. The number of calls received by a radio station during each hour of their afternoon schedule is displayed in the table below.

Time Period	Number of calls
2:00 p.m to 2:59 p.m.	150
3:00 p.m to 3:59 p.m.	142
4:00 p.m to 4:59 p.m.	160
5:00 p.m to 5:59 p.m.	356

a. Find the median number of calls per hour during the afternoon.

b. Find the mean number of calls per hour during the afternoon.

10. After taking five tests, Phil has earned a grade average of 93%. Phil misses the sixth test, and so he scores 0%. What is his new grade average after the sixth test?

Name _____ Class _____ Date _____

Lesson 5.1.4

Comparing Data Sets

California Standard: Statistics, Data Analysis, and Probability 1.1

Example

The range, mode, median, and mean of two data sets are shown below. Use these measures to compare the two data sets.

Set A: Range = 15, Mean = 9.5, Modes = 7 and 9, Median = 9.

Set B: Range = 7, Mean = 4, Mode = 3, Median = 5.

Solution

Set A has a higher typical value than set B, but its data is much more spread out.

1. The data set below shows the favorite colors of nine students in an art class.
{blue, red, green, pink, red, green, green, pink, green}

a. Does the set have a range? Explain. _____

b. Does the set have a median? Explain. _____

c. Does the set have a mean? Explain. _____

d. Does the set have a mode? Explain _____

2. The range and measures of central tendency for two data sets are as follows:

Set A: Range = 18, Mean = 10, Mode = 9, Median = 11.

Set B: Range = 18, Mean = 10, Mode = 9, Median = 11.

Since all the measures are the same, can you be sure that the data sets are identical?

_____

3. The city council wanted to know the ages of the people using a local park. They took two surveys, one on a Tuesday afternoon, and one on a Saturday morning. The range of ages they saw were:

Survey 1: Range = 40 years, Survey 2: Range = 70 years

Which survey is more likely to have been taken on Tuesday?

_____

4. Paula recorded the shoe sizes of all the sixth grade children on her school bus. This was the list she made: {4, 7, 6, 6, 9, 4, 6, 7}

a. Find the mode, median, mean, and range of Paula's data set.

_____

b. Alvar does the same on his bus. The range of his list is 8, his mean is 6, his median is 5, and his mode is 6. Use these measures to compare the foot sizes of Alvar's group to Paula's group.

_____

5. The two data sets below show the number of pets living in each of the houses on two streets.

Street 1: {2, 2, 3, 2, 0, 1, 2, 4} Street 2: {0, 2, 3, 5, 1, 1, 0, 1, 12, 3}

Say whether each of the statements below refers to street 1 or street 2.

a. The mean and the median of the data are the same. ----------

b. The data is spread across a large range. ----------

6. Calculate measures of central tendency and range for both of the data sets below.

Set 1: {1, 1, 4, 5, 7, 8, 9, 9, 10} Set 2: {4, 4, 5, 5, 5, 5, 5, 6, 6}

---

7. The two data sets below show the maximum outdoor temperature in °F over eight days in two towns.

Town A: {63, 64, 66, 70, 68, 70, 70, 73} Town B: {52, 52, 59, 60, 60, 59, 60, 62}

Find the mean temperature of both towns over the eight days. One of the towns is on the coast, and the other is on the edge of the desert. Which is most likely to be which? Explain how you can tell.

---

8. Miss. Williams calculated measures of central tendency and range for the test results of her Monday math group and her Thursday math group. The test scores of both groups are shown below.

Monday group: {27, 68, 63, 44, 88, 93, 75, 50, 77, 33, 90, 83, 40, 77, 82}

Thursday group: {72, 66, 75, 55, 77, 77, 68, 47, 70, 92, 60, 81}

a. Find the mode of both groups. ---------- ----------

b. Find the median score of both groups. ---------- ----------

c. Find the mean score of both groups. ---------- ----------

d. Find the spread of scores in both groups. ---------- ----------

e. Use the measures you have calculated in parts a–d to compare the two data sets.

---

9. Benton wants to find out whether the boys or the girls in his class go ten-pin bowling more often.

He asked the 10 boys and 12 girls in his class how many times they went bowling last month. His results are shown in the table on the right.

Find the mode, median, mean, and range of Benton's two data sets.

This means that 1 girl only went bowling once last month, while 4 boys bowled only once.

Times bowled last month	Number of girls	Number of boys
1	1	4
2	3	4
3	3	1
4	5	0
5	0	1

GIRLS: ----------

BOYS: ----------

Name _____ Class _____ Date _____

Lesson 5.2.1

Including Additional Data: Mode, Median, and Range

California Standards: Statistics, Data Analysis, and Probability 1.1, 1.2

Example

Mrs. Reed's history class got the following scores on a test: {65, 72, 80, 80, 80, 87, 87, 90, 92, 95}. She worked out that the range of their scores was 30, and the mode was 80. Then four more students, who had been absent the day of the test, took it. Their scores were 87, 87, 92, and 98. When Mrs. Reed added their scores to the list, what was the affect on the mode and the range?

Solution

New range = 98 – 65 = 33. **The range increased by 3, to 33.**
As two of the extra scores were 87, 87 is now the most common value in the data set, occurring four times. **The mode increased by 7, to 87.**

In this case, both the mode and the range changed. Sometimes, adding elements to a list might not change its central tendency or spread.

1. The data set below shows the ages of the children that came to a drama workshop.
{4, 7, 8, 8, 8, 9, 10}

a. Find the mode and the range of the children's ages.

b. Four children arrived late. Their ages were 6, 9, 9, and 12. Find the mode and range of the children's ages when the latecomers are included.

2. The data set below shows the colors of seven cars that went past a survey point.
{blue, red, silver, gold, red, green, silver}.

a. What is the mode of the set?

b. The surveyor realizes that he saw the same silver car twice. If one silver is removed from the set, what is the new mode?

3. Find the mode and range of the following data set: {0, 0, 1, 2, 3, 4, 4, 4, 5, 5, 6, 7}.
Now add the values 3, 5, 5, and 9 to the set. What are the mode and the range of the new set?

4. Find the mode and range of the following data set: {4, 7, 8, 8, 8, 9, 10, 10, 12, 15),
Now remove the values 8, 8, and 15 from the set. What are the mode and range of the new set?

5. The data set below shows the number of people traveling on four buses that pull into a bus station.
{30, 35, 37, 40}
A fifth bus pulls into the station. The number of people on this bus is added to the data set. Including it doubles the range of the set. What are the two possible numbers of passengers on the fifth bus?

6. Add three new values to this data set that do not change the mode: {4, 5, 5, 5, 5, 7}

---

7. Add three new values to this data set that do not change the range: {4, 5, 6}

Example

The data set below shows the rainfall, in inches, in six California towns in one year.
{3, 4, 5, 7, 9, 15} The median rainfall across these towns is 6 inches.
Two more towns with values of 13 in. and 17 in. are added to the set. How does this affect the median?

Solution

Put all the values from the new data set in order: {3, 4, 5, 7, 9, 13, 15, 17}
There are now eight values in the data set, so the median lies midway between the fourth and fifth values. The fourth value is 7, and the fifth value is 9, so the new median is 8. **The median increases by 2 inches, to 8 inches.**

The old median was between 5 and 7, and the new median is between 7 and 9.

8. This data set shows the number of toasters sold by an electrical store each day from Monday to Thursday of one week: {12, 8, 5, 10}. On Friday the store sold 12 toasters, and on Saturday they sold 13.

a. Find the median number of toasters sold from Monday to Thursday. ---

b. Find the median number of toasters sold from Monday to Saturday. ---

9. Pepe asks all of the students in his class who play musical instruments how many hours each week they spend practicing. These are the answers he got: {2, 6, 4, 8}.

a. What is the median of this data set? ---

b. Pepe gets two extra answers of 1 hour and 3 hours from people who were absent when he first took his survey. What is the new median of the set with these values added in? ---

10. Find the median of the following data set: {5, 8, 9, 11}.
Now add the values 12 and 12 to the set. What is the median of the new set?

---

11. The median of the following data set is 6: {3, 5, 7, 9}.
A new value, x, is added to the set. The median of the new set is still 6. What is x?

---

12. This data set shows the ages of children attending an art club: {6, 7, 7, 9, 12, 12, 12}. One of the children leaves the club when their family moves out of the area. When they leave, the median age of children attending the club drops from 9 to 8. What was the age of the child that left?

Name _____ Class _____ Date _____

Lesson 5.2.2

Including Additional Data: The Mean

California Standards: Statistics, Data Analysis, and Probability 1.1, 1.2

Example

The data set below shows the amount of snow that fell in seven towns during the month of January.
{37 inches, 25 inches, 22 inches, 36 inches, 44 inches, 29 inches, 52 inches}

a. Find the mean snowfall across the seven towns for that month.
Another town sends its snowfall data in late. It had a total of 71 inches of snow in January.

b. Predict what effect this new value will have on the mean.

c. Calculate the new mean snowfall with the extra value included.

Solution

a. Add all the values together: 37 + 25 + 22 + 36 + 44 + 29 + 52 = 245
Divide by the number of towns: 245 ÷ 7 = 35
So the mean January snowfall is **35 inches**.

b. The new value is much higher than the original mean value of inches.
So if you include the new value in your calculations, **the mean will increase.**

c. Add all the values together: 37 + 25 + 22 + 36 + 44 + 29 + 52 + 71 = 316
Divide by the number of towns: 316 ÷ 8 = 39.5
So the new mean January snowfall is **39.5 inches**.

1. The data set below shows the number of goals scored this season by members of a girls' soccer team.
{3, 3, 4, 4, 8, 9, 15, 17, 23, 29}

a. Find the mean number of goals scored. ----------

b. The goalkeeper very rarely has a chance to score, so her statistic is usually left off the list.
This season she did not score in any of the games. Find the new mean with her statistic included.

----

2. The local swimming pool runs two swimming clubs for sixth graders, one for learners and one for more experienced swimmers. The two data sets below show the times, in seconds, that it takes the swimmers in both clubs to swim a length of the pool.
Learners: {73, 77, 85, 78, 80, 90, 75, 70} Experienced: {45, 39, 41, 42, 46, 39, 42}

a. Calculate the mean length time for the learner swimmers. ----------

b. Calculate the mean length time for the experienced swimmers. ----------

c. Find the mean length time for all the sixth grade swim club members. ----------

3. Joe wants to score an average of 92% in his math tests, so he will get an A. On the first three tests, he scored 88%, 97%, and 83%. What score must he get on the fourth test to average 92%?

----

4. The data set below shows the number of points scored by a football player in six games.
{0, 6, 6, 3, 0, 9}

a. Find the mean number of points scored by the player.

b. In his next game, the player scores a total of four points.
What is his new mean score over the seven games?

5. The data set below shows the number of people who used a roadside phone booth each day, over five days.
{5, 7, 8, 5, 7}

a. Calculate the mean number of people that used the phone booth each day.

For the next three days, the road that the phone booth is on was closed, so no one used it.

b. Kelvin says that if he includes the three new 0 values in his calculation of the mean, it will not affect it. Is he right?

c. Calculate the mean number of people using the phone booth over all eight days.

6. Alicia is at a family reunion. She is talking to her cousins, whose ages are shown below.
{11, 12, 12, 14, 16}

a. Find the mean age of Alicia's cousins.

b. Alicia's Grandmother came over to join in the conversation. Alicia says that if she includes her Grandmother's age in the calculation of the mean, it increases to 25. How old is her Grandmother?

7. A new value, v, is added to the data set shown below.
{63, 74, 79, 84}
Once v is included, the mean of the set is 72. What is the value of v?

8. Jake made a note of his golf scores for the last seven rounds that he played. His scores are shown below.
{77, 72, 75, 78, 74, 69, 73}

a. What is Jake's mean golf score for the seven rounds?

b. The next two times that Jake played golf, he scored 79 and 78.
What is his new mean score after these two rounds?

c. On his tenth round of golf, Jake scored a 70.
Will adding in this value increase, or decrease, Jake's mean score? Explain.

Name _____ Class _____ Date _____

Lesson 5.3.1

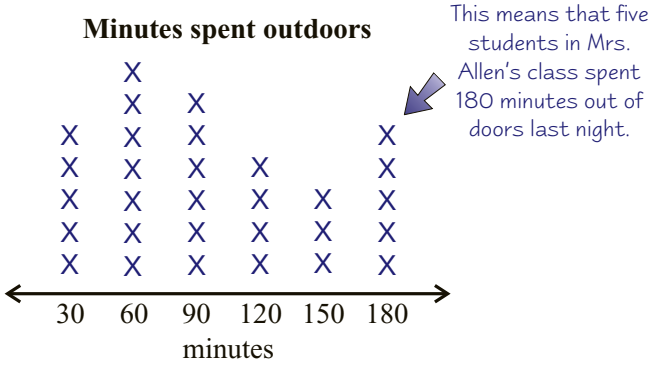
Analyzing Graphs

California Standards: Statistics, Data Analysis, and Probability 1.1, 2.3

Example

Mrs. Allen asked everyone in her class how many minutes they spent outdoors last night. Her results are shown on the line plot below.

Use the plot to find the mode and range of Mrs. Allen's data set.

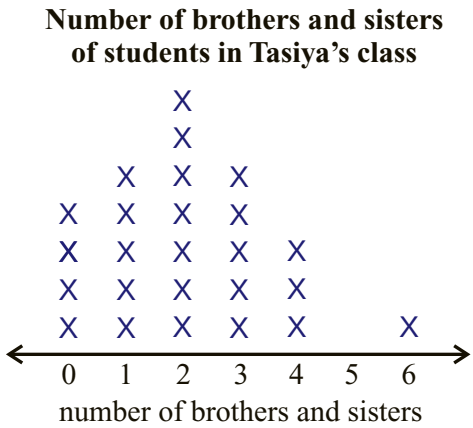


Solution

The range is the difference between the largest value and the smallest value in the data set. The highest value on the graph is 180 minutes, and the lowest is 30 minutes. So the range is 180 – 30 = **150 minutes**.

The mode is the most commonly occuring value in the data set. Of all the values on the graph, 60 has the most marks above it. So **60 minutes is the mode**.

1. Tasiya asked everyone in her class how many brothers and sisters they have. Her results are displayed on the line plot below.



a. How many students have exactly 4 brothers and sisters?

---

b. How many students have at least 3 brothers and sisters?

---

c. What is the mode of Tasiya's data set?

---

2. The pictograph below shows the number of students that regularly use each of five school bus routes serving a local high school.

Bus Route	Number of students
Route 1	4
Route 2	3.5
Route 3	5
Route 4	4.5
Route 5	4

Key: = 10 students

a. How many students regularly use bus route 4?

---

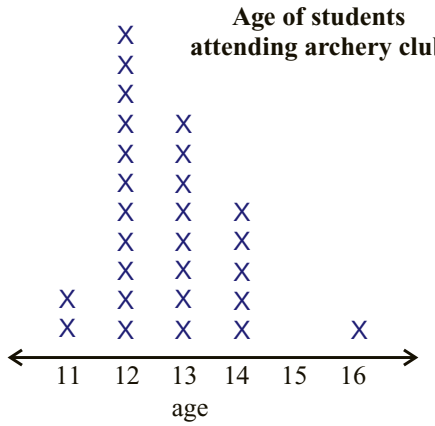
b. What is the range of the data set?

---

c. How many more students use bus route 5 than bus route 2?

---

3. The line plot below shows the ages of students that regularly attend an after school archery club at a local leisure center.



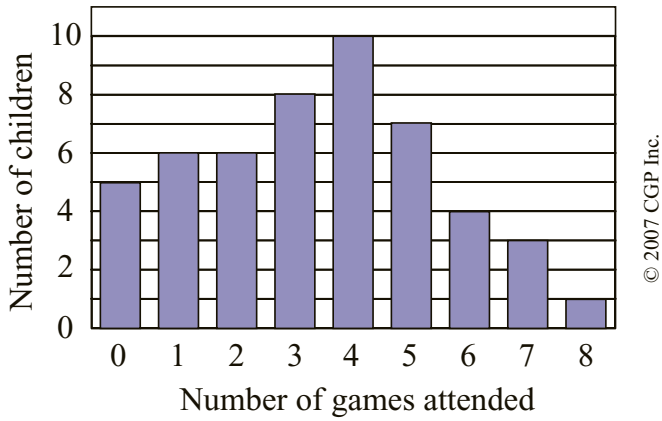
a. What is the age range of students attending the archery club?

---

b. What is the mode of the data set?

---

4. As part of a sports survey, 50 local children were asked how many times in the last year they had been to see the town's baseball team play. The results are shown on the bar graph below.



a. What is the range of the data set?

---

b. What is the mode of the data set?

---

c. How many children saw the town's baseball team play at least 5 matches in the last year?

Name _____ Class _____ Date _____

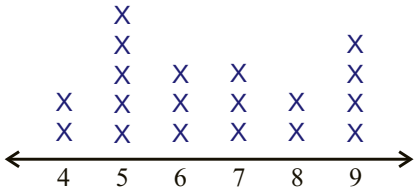
Lesson 5.3.2

Finding the Mean and Median from Graphs

California Standards: Statistics, Data Analysis, and Probability 1.1, 2.3

Example

What is the median of the data set shown on the line plot on the right?



Solution

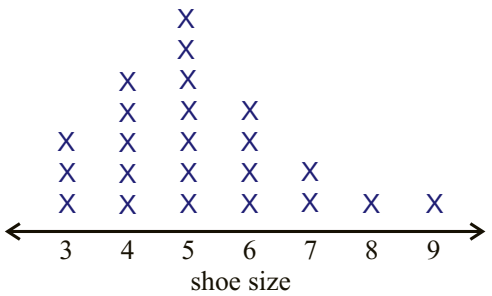
Each mark on the line plot represents one value from the data set that was used to construct it. So you can turn the plot back into a data set:

{4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 9, 9, 9, 9}

There are 19 values in the data set, so the median will be the 10th value.
The median is **6**.

1. The line plot below shows the shoe sizes of all the students in Mr. Taylor's math class.

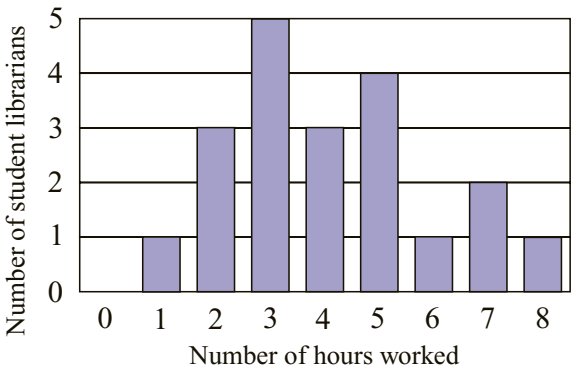
Shoe sizes of students in Mr. Taylor's math class



Find the median shoe size of students in Mr. Taylor's math class.

---

2. The bar graph below shows the number of hours worked each week by student librarians in the school library.

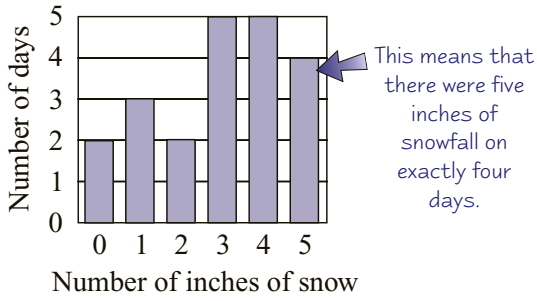


Find the median number of hours worked each week by the school library's student librarians.

Example

The bar graph on the right shows the number of inches of snowfall recorded each day at a California weather station over a 21 day period.

Find the mean of the data set shown on the bar graph using multiplication.



Solution

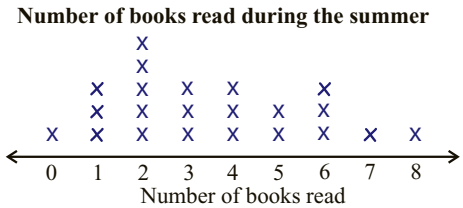
This data set has:

2 items with the value 0	$2 \times 0 = 0$
3 items with the value 1	$3 \times 1 = 1$
2 items with the value 2	$2 \times 2 = 4$
5 items with the value 3	$5 \times 3 = 15$
5 items with the value 4	$5 \times 4 = 20$
4 items with the value 5	$4 \times 5 = 20$
21	**60**

Sum the number of items, and the values

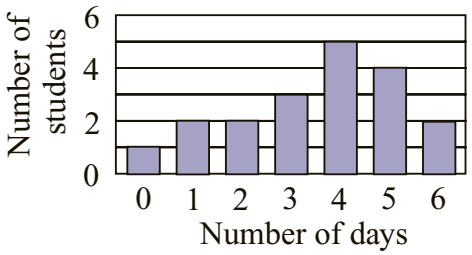
The sum of the values is 60, and there are 21 values.
So the mean is $60 \div 21 =$ **2.9 inches (to 1 decimal place)**

3. The line plot on the right shows the number of books read by all the students in one class over the summer vacation. Find the median and mean of the data set.



---

4. A group of 19 sixth grade students were asked how many days a week they participated in an organized sport outside of school. The results are displayed on the bar graph below.



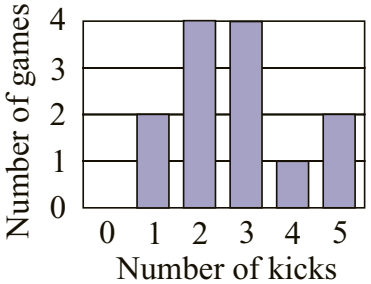
a. Find the median of the data set.

---

b. Find the mean of the data set.

---

5. The bar graph below shows the number of kicks made by a football kicker during his last 13 games.



a. What is the mean number of kicks made by the kicker, to the nearest hundredth?

---

b. In his 14th game, the kicker makes three kicks. What is the new mean of the data set, to the nearest hundredth?

Name _____ Class _____ Date _____

Lesson 5.3.3

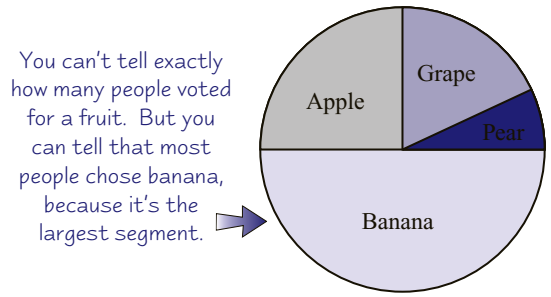
Other Types of Graphs

California Standard: Statistics, Data Analysis, and Probability 2.3

Example

Saskia asks the people on her school bus to choose their favorite fruit from a list of four. Her results are shown in the circle graph on the right.

List the fruits in order of popularity, starting with the most popular.



Solution

The part of the circle representing banana is the largest. The part of the circle representing apple is the second largest. The part of the circle representing grape is the third largest. The part of the circle representing pear is the smallest.

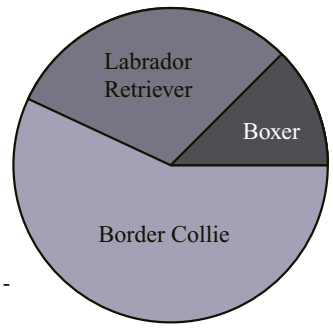
So in order of popularity, the fruits are: **banana, apple, grape, pear**.

1. The types of dogs owned by the members of a dog agility club are shown in the circle graph below.

a. Do more people at the club own Boxer dogs or Border Collies?

---

b. List the types of dog at the club in order from least common to most common.

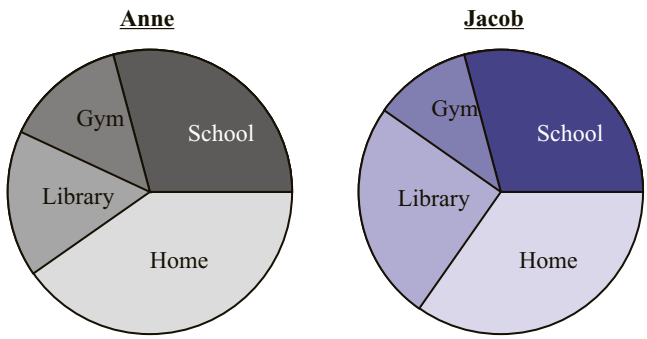


2. The two circle graphs to the right show the amount of time that Anne and Jacob spent yesterday at various different places.

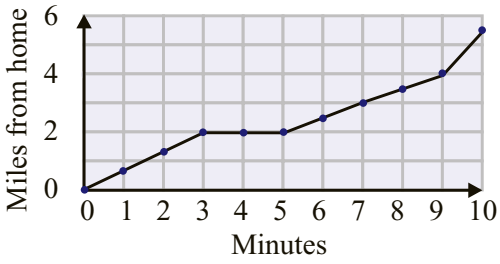
a. At which place did Anne spend more time than at the library, but less time than at home?

---

b. Which two places did Jacob spend less time at than Anne yesterday?



3. The line graph on the right shows the distance that Bryah was from home as she rode the bus to school.



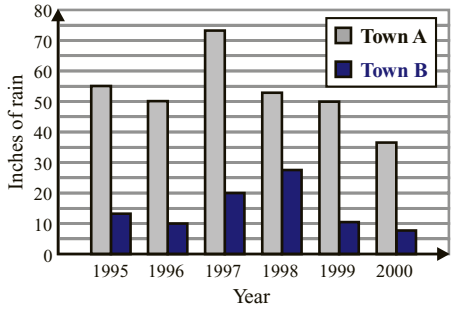
a. How far from home was Bryah after 7 minutes?

---

b. The bus was stuck in a traffic jam for 2 minutes. Looking at the graph, can you say when this was? Explain.

Example

The graph on the right shows the average annual rainfall in Town A and Town B between 1995 and 2000.

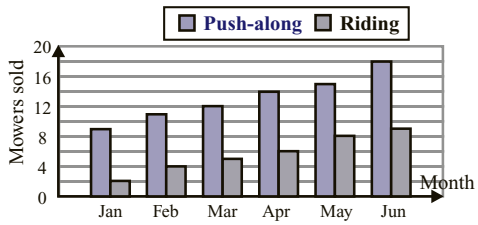


What is the difference between the rainfall received in Town A and Town B in 1996? Does Town A or Town B usually have the higher annual rainfall?

Solution

Read the rainfall values for both towns in 1996 from the graph: Town A = 50, Town B = 10.
Now find the difference: 50 – 10 = **40 inches**.
You can see from the graph that **Town A** usually has the higher annual rainfall, because all the bars representing Town A are taller than the bars representing Town B for the same year.

4. The bar graph to the right shows the number of push-along lawn mowers and riding lawn mowers sold by a garden center over six months.



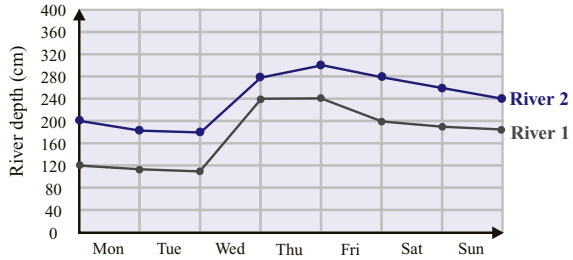
a. How many of each type of mower did the center sell in April?

---

b. What is the range of the riding lawn mower data set? ---

c. How many more push-along than riding mowers were sold in June? ---

5. The line graph to the right shows the depth of two nearby rivers over the course of a week. The depth of the river is recorded at midnight each day.



a. What was the difference in the depths of the rivers when they were measured at the start of Saturday?

---

b. On one day of the week, it rained very heavily. Explain which day you think this was.

Name _____ Class _____ Date _____

Lesson 5.4.1

Using Samples

California Standards: Statistics, Data Analysis, and Probability 2.1, 2.4

Example

The Principal wants to ask students their opinions about the new school cafeteria menu. The school has 1550 students, so the Principal decides to ask a sample of students. What does he mean by this?

Solution

There are too many students in the school to ask every single one for their opinions. So the Principal will select a number of students to ask, and assume that their answers will be representative of the opinions of all the students at the school.

1. A candy company checks every fifth bar it makes before packing to make sure that it is still properly wrapped.

a. What is the population of this survey? _____

b. Identify the sample used for this survey. _____

2. The data set below shows the number of hours that a group of students spent playing sports last week.
{2, 4, 4, 5, 5, 5, 6, 6, 6, 6, 7, 7, 8, 8, 8, 9, 9, 9, 10, 10}
Explain why each of the following is not a sample of this data set.

a. {2, 4, 5, 6, 7, 8, 9, 10, 12} _____

b. {2, 4, 4, 5, 5, 5, 6, 6, 6, 6, 7, 7, 8, 8, 8, 9, 9, 9, 10, 10} _____

c. The median, which has a value of 6.5. _____

3. A train company wants to conduct a survey to find out whether its passengers are happy with the cleanliness of its trains. Suggest a sensible way for them to choose a sample of passengers to survey.

_____

4. A researcher wants to gather information about the nesting sites of California snow geese. He captures some of the birds at various sites around the state, fits identification rings to their legs, and releases them back into the wild, where their movements can be tracked.

a. What is the population of the researcher's survey? _____

b. What sample has the researcher used? _____

5. In the surveys described below, say whether you would collect data from the whole population, or from a sample.

a. Finding the mean height of the students in your math class. ----------

b. Finding how much is recycled each month by California households. ----------

c. Finding the typical cost of a pint of milk in the U.S.A. ----------

Example

A sports company wants to find out how often the typical American goes surfing, so they send out a watersports questionnaire to 5000 randomly selected Californians. Explain why this study is biased.

Solution

The aim of the study was to find out how often the typical American goes surfing.
But the questionnaire was only sent to Califonians, so the rest of the population of the U.S.A. is unrepresented, and the study is biased.

6. Warren is conducting a survey. He wants to find the favorite subject of students in his school. He asks everyone in the drama club what their favorite subject is.

a. What are the population and the sample of Warren's survey?

---

b. Explain why Warren's sample is biased.

---

c. Suggest a more sensible sample for Warren to use.

---

7. Company A wanted to find out whether customers at a certain store prefer the washing powder they make to those made by their rival companies. They asked the store to give a product questionnaire to anyone buying their product, asking if it is the washing powder that they usually buy.

a. What were the population and sample of the survey?

Of the questionnaires that were returned, three-quarters said that they did usually buy Company A's product. Representatives of Company A then claimed that their washing powder was preferred by 75% of the store's customers.

b. Is this a valid claim? ----------

c. Explain your answer to part b.

---

d. Suggest how a more representative sample of store customers could be chosen for this study.

Name _____ Class _____ Date _____

Lesson 5.4.2

Convenience, Random, and Systematic Sampling

California Standards: Statistics, Data Analysis, and Probability 2.2, 2.4

Example

Elesha, Matt, and Luiza want to know how far away from their school the average student lives. They decide to ask a sample of the population of students. Elesha asks all her friends. Matt makes a list of everyone at the school, has a computer select 40 names from the list at random, and asks those students. Luiza stands at the school door and asks every 40th student that comes in.

What sampling methods have Elesha, Matt, and Luisa used to find their samples?

Solution

Elesha found a group of people who were easily available to her to ask — her friends.
So she used **convenience sampling**.

Matt asked a group of people that he selected completely at random.
So he used **random sampling**.

Luiza chose people for her sample using a set pattern.
So she used **systematic sampling**.

1. Say whether each of the methods described below is an example of convenience sampling, systematic sampling, or random sampling.

a. You want to know the most popular car color in your town. So you stand on the corner of your street and record the different colors of the first 50 cars that go by.

---

b. A store manager wants to know how much his customers spend on average each time they visit the store. He places a sales clerk at the doors to the store and has them question every fifth shopper.

---

c. Calvin wants to find how much the average American household spends each month on food. He asks all the families that live on his street.

---

d. Jen wants to know the height of a typical sixth grader at her school. She makes a list of all the sixth graders, giving each one a number. Jen puts numbered chips in a bag, and pulls out 20. She measures the height of the 20 students whose list numbers correspond to the numbers of the chips.

---

2. Cid wants to find the average age of people using the town library on a Saturday. He stands in the children's section, and asks the ages of the first twenty people to pass him.

a. What sampling method was Cid using? ---

b. Explain why his sample may be biased. ---

3. A machine making pens at a factory develops a fault and begins to put the wrong colored ink in every fifth pen. There are two quality control inspectors checking the pens as they come off the machine.

a. The first inspector is checking every thirteenth pen, starting at pen number thirteen. How many pens will he check before he finds a faulty one?

---

b. The second inspector checks every fifth pen, starting with pen number three. How many pens will he check before he discovers a faulty one?

Example

The editor of the school paper asks you to write an article on what improvements students would like to see in the cafeteria lunch program. You make up a survey to gather information. How could you choose a systematic sample of students to answer your survey?

Solution

Any method of selecting students to answer the survey that relies on a set pattern will give you a systematic sample. For example, you could stand at the school door in the morning and question every tenth student that comes past you. Or you could take a list of the names of all the students at the school and go and interview every twentieth person.

Random sampling must give each member of the population an equal chance of being selected.

4. You want to find the average number of books read by the 25 students in your class over the summer. How could you select a random sample of 8 students from your class to ask?

---

5. You want to find out how many customers at your local store usually buy a daily newspaper.

a. What is the population of your survey?

---

b. Give an example of how you might use convenience sampling to select a sample for your survey.

---

6. Jane is the secretary of her town's athletics club. She wants to know if the club members would like the town to stage a yearly marathon event. Give an example of a way that Jane could pick a systematic sample of club members to ask.

---

7. The student council wants the students' opinion on whether or not a fruit juice kiosk should be put in the main hall of the building. They ask Max, a sixth grader on the council, to take a survey. Max stands in the middle of sixth grade block stopping people and asking their opinion.

a. What kind of sampling was Max using?

---

b. What bias might this introduce to Max's sample?

---

c. Give an example of a way that Max could have picked a random sample instead.

Name _____ Class _____ Date _____

Lesson 5.4.3

Samples and Accuracy

California Standards: Statistics, Data Analysis, and Probability 2.1, 2.4

Example

Katia asked everyone on her street how many pets they had. The data she collected is shown below.
{2, 2, 0, 5, 3, 2, 3, 0, 3, 2, 1, 4, 1, 2, 0}
Katia took the following sample from her data set, and found the sample mean: {1, 2, 2, 3, 3, 4}.
Find the sampling error of the mean that Katia calculated.

Solution

First find the mean of the population:
$2 + 2 + 0 + 5 + 3 + 2 + 3 + 0 + 3 + 2 + 1 + 4 + 1 + 2 + 0 = 30$ **Add all the values**
$30 \div 15 = 2$ pets **Divide by the number of values**

Find the mean of the sample: $1 + 2 + 2 + 3 + 3 + 4 = 15$
$15 \div 6 = 2.5$ pets

Now find the sampling error: $2.5 - 2 =$ **0.5 pets** ← The sampling error is the difference between the population mean and the sample mean.

1. The data set below shows the number of baskets scored by members of a basketball club in one evening's practice.
{3, 8, 7, 9, 10, 4, 6, 11, 5, 9, 7, 6, 5, 4, 8, 6, 9, 5, 11, 3}

a. Calculate the mean number of baskets scored per member. _____

Find the sampling errors for the mean if the following samples are used.

b. {3, 5, 7, 9, 11} _____

c. {3, 4, 5, 6, 11} _____

2. The data set below shows the ages of the members of a hiking club.
{15, 21, 16, 18, 17, 20, 16, 19, 25, 22, 20, 13, 19, 21, 20}

a. Find the median age of the members of the club. _____

Find the sampling errors for the median if the following samples are used.

b. {15, 17, 18, 20, 22} _____

c. {19, 20, 22, 22, 25} _____

3. Marcus asked everyone sitting in the park how many times in an average week they came there. These are the answers he got: {7, 11, 13, 8, 7, 9, 12, 6, 9, 10, 7, 9}

a. What is the population mean? _____

b. Marcus selected the following sample from his data set: {7, 8, 12, 10, 9}. What is the sample mean? _____

c. Find the sampling error for the mean of Marcus's sample. _____

d. Pick your own sample of 6 values from the original data set. Determine the mean and sampling error of the mean for your sample. _____

4. The data set below shows the times, in seconds, of all the competitors in a 100 meter race.
{12.2, 11.8, 11.4, 11.9, 11.5, 11.0, 11.3, 11.1, 11.0, 11.0, 11.1, 10.9, 10.5, 10.8, 10.9, 10.8}
Javan took the following sample from the data set: {11.9, 11.3, 11.1, 11.0, 10.8, 10.8}.

a. What is the mean of the population?

---

b. What is the median of the population?

---

c. What is the sampling error of the mean for Javan's sample?

---

d. What is the sampling error of the median for Javan's sample?

---

e. Is Javan's sample a good representation of the population?

---

5. The data set below shows the scores of 20 students that took a math exam.
{178, 189, 201, 222, 244, 244, 252, 260, 263, 270, 279, 288, 299, 299, 306, 315, 323, 347, 366, 377}

a. Take a sample of 6 items from the set. Calculate the sampling error of the mean for your sample.

---

b. Take a sample of 10 items from the set. Calculate the sampling error of the mean for your sample.

---

c. Was the sampling error smaller for the first or second sample?

---

6. Francesca collected information about the annual rainfall received by three towns for the last fifty years. The table on the right shows the average annual rainfall received by all three towns over those fifty years, and also the means for a sample that Francesca took from each data set.

Find the sampling error of the mean for all three towns, and say for which town the sample comes closest to matching the characteristics of the data set.

	Mean Average Rainfall (cm)	Sample Mean (cm)
Town A	27	26.3
Town B	54	49.6
Town C	102	104.6

TOWN A: ---------- **TOWN B:** ---------- **TOWN C:** ----------

---

7. A data set was randomly sampled three times. The median of the population and the median of each sample is shown in the table below. Find the sampling error of the median for all three samples. Which sample is the best reflection of the whole population?

Population Median	Sample A Median	Sample B Median	Sample C Median
77	75	75.5	79

Name _____ Class _____ Date _____

Lesson 5.4.4

Questionnaire Surveys

California Standards: Statistics, Data Analysis, and Probability 2.2, 2.3

Example

A mall wants to know if its customers are happy with the amount of parking it provides. Copies of a parking questionnaire and a box to put responses in are left in the main entrance. What are the population and the sample of this survey? Explain some of the benefits and problems of collecting data in this way.

Solution

The population of the survey is all the mall's customers. The sample is those customers who notice, pick up, answer, and return the questionnaires.

Benefits: the mall will learn some information about customer's opinions of its parking facilities.

Problems: the sample is self-selecting — it is likely that only those customers with strong opinions will bother to respond, leading to a biased sample. Also, as the mall is relying on its customers to choose to answer the survey during their time at the mall, they may not get enough responses to be able to draw any useful conclusions.

1. A school has the money to fund a new after-school club, and wants to know if students would prefer a languages club or an art club. It decides to take a survey. Suggest whether each of the following sample selection methods would result in a biased sample, and if so, explain why.

a. Stand outside the art department and question every tenth student that passes.

---

b. Pick 40 names completely at random from the school register and go and ask their opinion.

---

c. Explain to all the students what the survey is, and ask for two volunteers from each class to answer it.

---

2. A new coffee shop is opening in the high street. Free coffee is given to the first 100 people who walk by. They are then asked their opinion of the coffee. Is this a biased sample? Explain.

---

3. A local sports radio station wants to know if city residents are in favor of a plan to build a city skate park. They ask people to call in and vote yes or no to the skate park. Explain some of the benefits and problems of collecting their data in this way.

---

4. A company wants to know what consumers think of its cereal. They put a message on the box asking customers to complete an online survey. They also offer to send a free box of cereal to everyone that completes it. Explain some of the benefits and problems of collecting their data in this way.

Example

Dexter wants to know how the students at his school feel about the cafeteria menu. He stops students at random, and asks them the following question: "Do you like the cafeteria's excellent and very nutritious menu?" Why is this question biased? Rewrite the question so that it is fair.

Solution

The question is biased because it gives a strong positive opinion about the cafeteria food — making it difficult for students to disagree. A fairer question would be something like this: "What do you think of the food that is served in the school cafeteria?"

5. Decide whether or not each of the following survey questions is biased. If so, explain why.

a. Should ordinary taxpayers be asked to pay for a new stadium for a rich professional football team?

---

b. Should our school have a uniform?

---

c. Do you think we should make our public parks safer by banning motorized scooters?

---

6. Rewrite the biased questions from the last exercise as fair questions.

---

7. A school wants to take a survey to find out if students would like more trash cans on the schoolyard.

a. Write a survey question that is biased in favor of having more trash cans.

---

b. Write a survey question that is biased against having more trash cans.

---

c. Write an unbiased question for the survey.

---

8. A researcher stops 100 people on the street, and asks them this question: "Do you support the city council's scheme to improve our children's health by building more sports facilities?" He reports back that 80% of the city's residents support the scheme. Is this a valid claim? Explain.

---

9. A company wants the city council to ban cars from the city center. They stop 200 people at random in the city center, and ask them the following questions, with the following results:

1) Are you a worker at, or a customer of, a city center business? Yes: 97%, No: 3%
2) Do you feel that road safety is an issue that is relevant to you? Yes: 90%, No: 10%
3) Are you for or against cars being allowed in the city center? For: 25%, Against: 75%

They tell the council that there is 75% support for their plan. Explain how their survey was biased.

Name _____ Class _____ Date _____

Lesson 5.5.1

Evaluating Claims

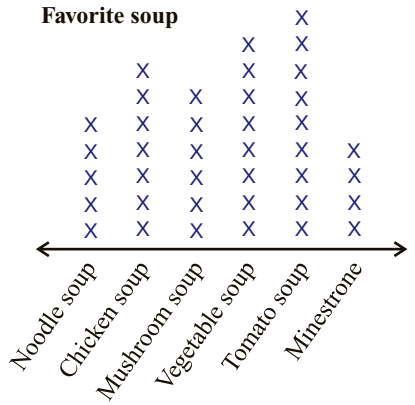
California Standard: Statistics, Data Analysis, and Probability 2.5

Example

The school newspaper says that chicken soup is the most popular soup flavor among students in the lunch room.

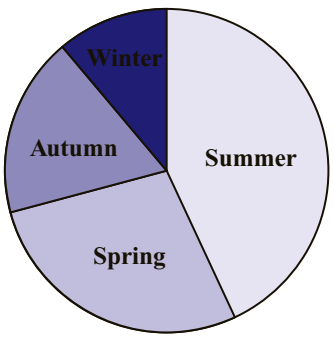
To determine the validity of this claim, Lilia questions 40 randomly selected students in the lunch room. The line plot on the right shows her results.

Does Lilia's line plot support the school newspaper's claim?



Solution

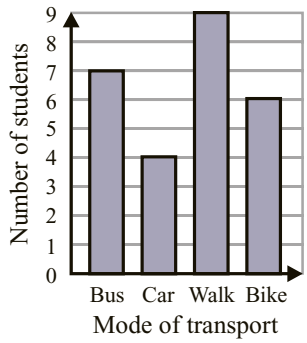
Out of 40 randomly selected people. 7 people chose chicken soup as their favorite flavor. But 9 people chose tomato soup, and 8 people chose vegetable soup. So, in Lilia's sample, chicken soup is the third favorite soup flavor, with tomato soup being the favorite.
Lilia's line plot does not support the school newspaper's claim.

1. 

The owner of a sportswear store claims that his store sells more swimwear in the summer than in any other season. Does the circle graph on the left, which shows what proportion of the store's swimwear sales are in each season, support his claim? Explain your answer.

---

2. Mike asks everyone in his class how they usually come to school. The data he collected is shown in the bar graph on the right. Mike claims that most people in his class walk to school. Does the graph support his claim? Explain your answer.



Example

The table on the right shows the number of inches of snow that fell on two California ski resorts over one week in February.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Resort A	6	6	6	13	3	2	6
Resort B	7	8	9	16	3	3	3

Resort A claims to have more snow on a typical day than Resort B. Does the data support this claim?

Solution

The modal number of inches for Resort A is 6 inches. The modal number of inches for Resort B is 3 inches. So this does support Resort A's claim.

Putting both data sets in order: {2, 3, 6, 6, 6, 6, 13} {3, 3, 3, 7, 8, 9, 16}

The median number of inches for Resort A is 6 inches, and the median number of inches for Resort B is 7 inches.

The mean number of inches for Resort A was: $(2 + 3 + 6 + 6 + 6 + 6 + 13) \div 7 = 42 \div 7 = 6$ inches

The mean number of inches for Resort B was: $(3 + 3 + 3 + 7 + 8 + 9 + 16) \div 7 = 49 \div 7 = 7$ inches

If you use any other measure of central tendency than the mode, Resort B's typical value is higher.

So you could say that Resort A's claim was true if you were considering only the mode, but it is misleading.

3. The following data set shows the number of minutes that ten players played during a basketball game.

{6, 10, 13, 20, 23, 25, 25, 25, 31, 32}

a. Find the mode, median, and mean of the data set.

Use the measures of central tendency you calculated in part a to say whether each of the claims below is valid or not. Explain your answers.

b. Most of the players played for 25 minutes.

c. Most of the players played for at least 20 minutes.

d. The typical number of minutes spent on court per player was 21.

4. The table below shows the number of people that visited three stalls at the town festival over the four days that it was on.

Stall	Wednesday	Thursday	Friday	Saturday
Coconut shy	150	142	200	400
Toy stall	224	204	370	844
Tea stand	310	296	440	798

a. The toy stall claims to be the most popular of the three stalls at the festival. Give a piece of evidence to support its claim.

b. The tea stand also claims to be the most popular of the three stalls at the festival. Give a piece of evidence to support its claim.

Name _____ Class _____ Date _____

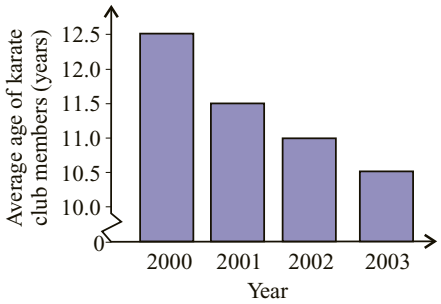
Lesson 5.5.2

Evaluating Displays

California Standard: Statistics, Data Analysis, and Probability 2.3

Example

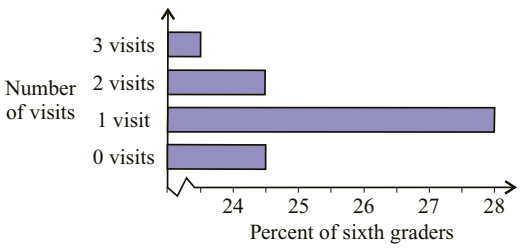
Sally claims that the average age of the members of her karate club has decreased dramatically over the last four years. She uses the graph on the right to support her claim.



Explain why someone might complain that Sally's graph is misleading. How much has the average age of karate club members actually changed by over the four years shown?

Solution

The break in the vertical scale makes the change in the height of the bars look much more dramatic than the small age change it represents. If the vertical scale were to start at 0, the difference would appear to be much less dramatic. The actual decrease in average age was 12.5 – 10.5 = **2 years**.

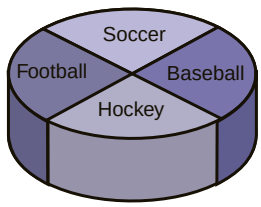
1. 

Devin takes a survey to find how many times the sixth graders visit the mall in a typical month. He drew the graph on the left to show his results.

Devin says that many more sixth graders make one visit to the mall in a typical month than make zero, two, or three visits. Is his claim valid? Explain.

---

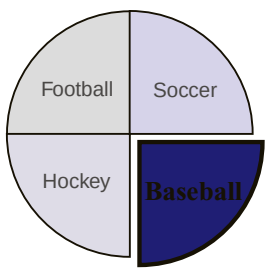
2. Fred asked all the students in his class what their favorite sport was. The results of his survey are shown in the circle graph on the right.



Explain why someone might complain that the graph is misleading.

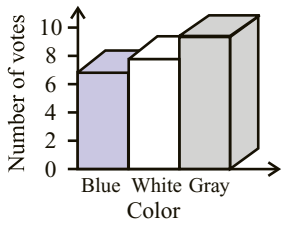
---

3. Savannah is a baseball fan. She redraws Fred's sports circle graph like this:



Explain two ways in which Savannah's graph is misleading.

4.



The members of Ben's soccer club are deciding what color their new jerseys should be. Ben asks everyone in the club for their opinion. Then he makes the graph shown on the left to give to the club committee.

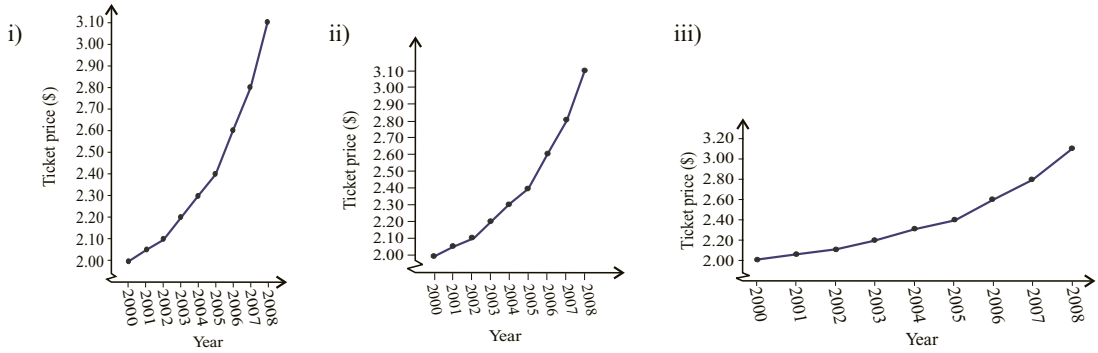
a. In what way is Ben's graph misleading?

---

b. Suggest a way that he could change the graph to make it less misleading.

---

5. These graphs all show the change in the cost of a bus ticket on a certain route over an eight year period.



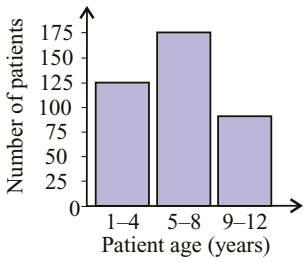
a. Which of the graphs makes the price appear to rise most dramatically? Explain your answer.

---

b. If you were making an advertisement for the bus company showing how their prices had stayed low for several years, which graph would you use? Explain your answer.

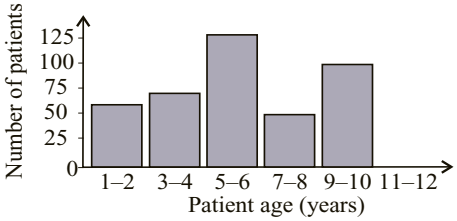
---

6. The graph on the right shows the number of patients of different ages that were seen by a pediatrician over the course of one year.



a. What is the age interval for which he saw the greatest number of patients?

The graph below shows the same data, but split into different intervals.



b. How are the intervals on the second graph different?

---

c. Why might the intervals on the second graph be a better choice in this case?

Name _____ Class _____ Date _____

Lesson 6.1.1

Listing Possible Outcomes

California Standard: Statistics, Data Analysis, and Probability 3.1

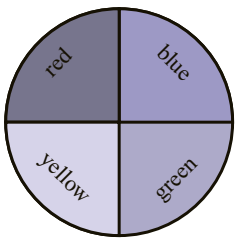
Example

Janek has a 6-sided die numbered 1, 2, 3, 4, 6, 6.
How many possible outcomes are there when he rolls the die?
List all the possible outcomes.

Solution

Each side of the die represents one outcome.
So there are **6 possible outcomes**: **1**, **2**, **3**, **4**, and two outcomes of **6**.

1. What are the possible outcomes when the spinner shown is spun?



---

2. A spinner has 5 sections colored red, green, red, yellow, and green.
How many possible outcomes are there when spinning the spinner?

---

3. Dave and his friends use the spinner shown to choose which sport to play. What are the possible outcomes?

---

4. A 9-sided die numbered 0, 1, 1, 2, 3, 4, 5, 6, 7 is rolled.
List all possible outcomes that match the following events.

Remember — 1 isn't a prime number.

a. Event: rolling a prime number. _____

b. Event: rolling a number that is not prime. _____

c. Event: rolling a number less than 4. _____

d. Event: rolling a number greater than 7. _____

e. Event: rolling a number less than 8. _____

f. Event: rolling a negative number. _____

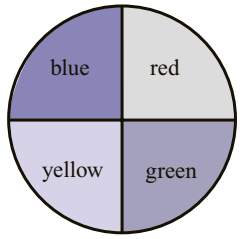
Example

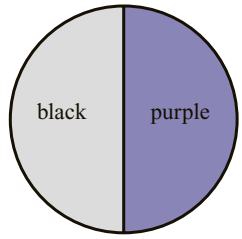
Letitia tossed a coin twice.
List all the possible outcomes.

Solution

The possible outcomes are:
"**head and head**," "**head and tail**," "**tail and head**," and "**tail and tail**."

5. Keira and Rob spin the two spinners shown.
If one possible outcome of these spins is "blue and black," list all the other possible outcomes.





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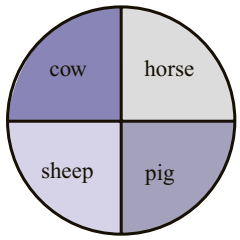
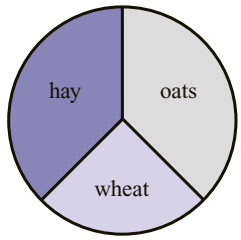
6. Paula rolls a normal 6-sided die, while Ashley spins a spinner with four sections, numbered 1, 2, 3, 4.
What are the possible outcomes of this combined action?

---

7. Jerry spins the two spinners shown at the same time.
List all the possible outcomes.





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8. Andy tosses a quarter 3 times. How many possible outcomes of this combined action are there?
List all the possible outcomes.

---

9. Christina rolls a normal 6-sided die, while Trevor spins a spinner with 12 sections, labeled with the letters A through L. How many different "number and letter" outcomes exist?

---

10. A.J. and Dorian each roll a normal 6-sided die at the same time. They combine their results to form a 2-digit number, where A.J.'s roll is the tens digit and Dorian's number is the ones digit.
How many 2-digit numbers can they make?

Name _____ Class _____ Date _____

Lesson 6.1.2

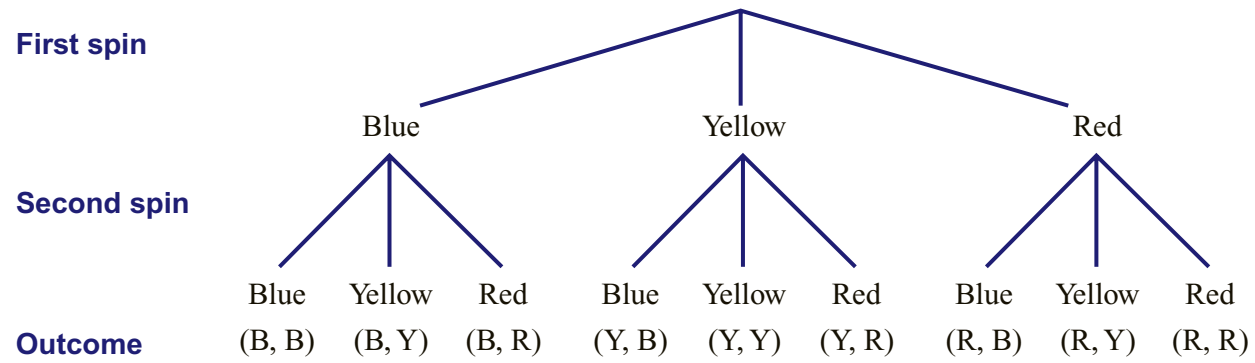
Tree Diagrams

California Standard: Statistics, Data Analysis, and Probability 3.1

Example

What are the possible outcomes if a spinner with red, yellow, and blue sections is spun twice? How many possible outcomes exist?

Solution

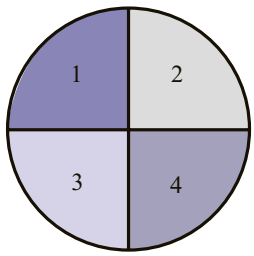


The tree diagram shows all possible outcomes of the two spins.
There are 9 possible outcomes.

1. Two spinners, one labeled 1, 2, 3, and the other colored red and yellow, are spun at the same time. Draw a tree diagram showing all the possible outcomes of these actions.

2. Draw a tree diagram to show the outcomes when rolling a normal 6-sided die and then flipping a coin.

3. Ricardo spins the spinner shown twice. Draw a tree diagram to show all the possible outcomes.



4. Lisa pulls one of the lettered tiles shown out of a bag, then flips a coin. Draw a tree diagram to show all the possible outcomes.

5. The local deli makes sandwiches using 2 types of bread (wheat and white), and 3 choices of filling (turkey, ham, and salami). Draw a tree diagram to show all the different combinations of bread and filling that are possible.

6. Darlene is looking at new cars. The garage has 2 models (X and XV), 3 colors (white, silver, and red), and 2 different interior colors (gray and black). Draw a tree diagram to show all the possible combinations.

7. Samuel puts three counters, numbered 1, 2, and 3, into a bag. He uses the counters to write a 3-digit number by drawing a counter, writing its number down, then replacing it. He does this three times altogether to find the three digits. Draw a tree diagram to show the different 3-digit numbers that Samuel could write.

8. Tion has one spinner labeled with the numbers 1 and 2, and another spinner labeled with the numbers 4, 5, 6, and 7. He makes a 2-digit number by spinning both spinners and using the result from the 2-part spinner as the tens digit, and the result from the 4-part spinner as the ones digit. What numbers can Tion make? Draw a tree diagram to show all the possibilities.

Name _____ Class _____ Date _____

Lesson 6.1.3

Tables and Grids

California Standard: Statistics, Data Analysis, and Probability 3.1

Example

Greg has two spinners. One spinner is colored red, blue, and green, while the other is colored pink, orange, yellow, and white. Using a grid, show all the possible color combinations when Greg spins both spinners.

Solution

Possible outcomes of each spinner

	Pink	Orange	Yellow	White
Red	RP	RO	RY	RW
Blue	BP	BO	BY	BW
Green	GP	GO	GY	GW

Color combinations, for example: BW ⇒ blue and white

1. Marshall is packing to go on vacation. He packs green, red, white, and yellow shorts. He also packs tennis shoes, boots, and sandals. Using a table, show all the possible combinations of shoes and shorts that Marshall can wear on vacation.

2. Keris is creating a character in a computer game. There are 4 choices for the color of hair, and 4 choices for the height. How many different ways are there to combine hair color and height? Show these combinations in a table.

---

3. Mrs. Jones is trying to determine the best combination of laundry soap and bleach. If she has 2 types of soap and 3 types of bleach, how many different combinations must Mrs. Jones try? Draw a grid to show all the combinations.

4. Two dice are rolled and the results multiplied together.
Draw a table to show all the possible outcomes.

5. Two numbered spinners are spun. One spinner is labeled with the numbers 3, 6, 10, and 14, while the other is labeled with 2, 4, and 6. The table shows all the possible outcomes of adding the results from the two numbered spinners. Write down all the possible outcomes that match the following events.

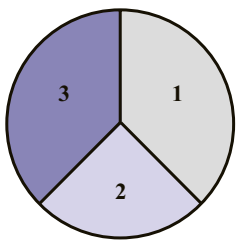
	3	**6**	**10**	**14**
2	5	8	12	16
4	7	10	14	18
6	9	12	16	20

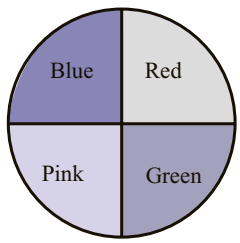
a. The sum ends in 5. ----------

b. The sum matches two different outcomes. ----------

c. The sum is a multiple of both 3 and 4. ----------

6. Construct a grid showing all possible outcomes when the 3-part spinner and the 4-part spinner shown are spun.





How many possible outcomes are there?

---

7. Marcus has a 5-section spinner, a 6-sided die, and a 4-sided die.
How many possible outcomes are there when he uses all three? Explain your answer.

Name _____ Class _____ Date _____

Lesson 6.2.1

Probability

California Standard: Statistics, Data Analysis, and Probability 3.3

Example

State whether the probability of each of the following events is 0, 0.5, or 1.
a. The result of rolling a normal die is less than 4.
b. The result of rolling a normal die is 7.
c. The result of rolling a normal die is either even or odd.
d. The result of tossing a normal coin is "heads."
e. A spinner divided into equal red and blue sections stops on blue.

Solution

a. 0.5 **The result is equally likely to be less than 4, or not less than 4**
b. 0 **It's impossible to get 7 on a normal die**
c. 1 **All numbers on a die are either even or odd, so this result is certain**
d. 0.5 **This result is equally likely to happen and not happen**
e. 0.5 **The spinner has an equal chance of landing on red or blue**

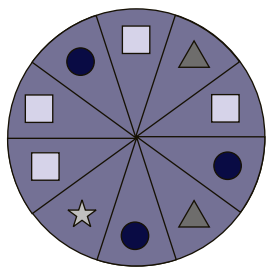
1. Write each of the following probabilities as a decimal.

a. 35% _____

b. 25% _____

c. 5% _____

2. Give the probability of spinning each shape, as a fraction.



a. A star _____

b. A circle _____

c. A square _____

d. A triangle _____

3. A magician has 7 red, 8 yellow, and 5 blue scarves in a hat. What is the probability of the magician pulling out each color of scarf if he picks one out at random? Give your answers as fractions.

a. Red _____

b. Yellow _____

c. Blue _____

4. Kendra has 20 blue chips in a bag. She reaches in and draws one chip.
If there are no other chips in the bag, what is the probability that:

a. the chip is blue? ----------

b. the chip is green? ----------

5. Estimate the probability of the following events.

a. It will rain when there are no clouds to be seen.

---

b. The Sun will rise in the morning.

---

c. A 5 comes up when rolling a die with 5s on all the sides.

---

d. A 3 comes up when rolling a die with 5s on all the sides.

---

e. You will play some kind of sport this weekend.

---

f. You will watch T.V. tonight.

---

6. A spinner has 30 equally-sized colored sections.
How many of the sections must be black for the following probabilities to be correct?

a. Probability of spinning black = 1 ----------

b. Probability of spinning black = 0 ----------

c. Probability of spinning black = $\frac{1}{2}$ ----------

Name _____ Class _____ Date _____

Lesson 6.2.2

Expressing Probability

California Standard: Statistics, Data Analysis, and Probability 3.3

Example

A bag contains 8 red chips and 6 blue chips. Karl reaches into the bag and draws one chip at random. What is the probability of the following events?

a. The chip is red. Give your answer as a fraction.

b. The chip is blue. Give your answer as a decimal.

c. The chip is green. Give your answer as a percent.

Solution

$$\text{P(event)} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

a. $\text{P(red)} = \frac{8}{14} = \frac{4}{7}$ — There are 14 chips in total and 8 are red

b. $\text{P(blue)} = 6 \div 14 = \mathbf{0.43}$ — There are 14 chips in total and 6 are blue

c. P(green) = **0%** — None of the chips are green so this event is impossible

1. The probability of an event is $\frac{5}{8}$. Write this probability in two other ways.

---

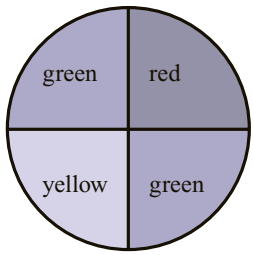
2. Give the probability of the following events as a fraction, a decimal, and a percent.

a. Spinning red ---

b. Spinning green ---

c. Spinning yellow ---

d. Spinning blue ---



3. A bag contains 4 blue chips, 5 green chips, and 6 black chips.

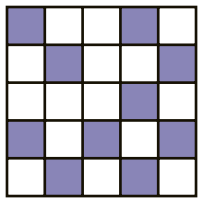
a. How many chips are in the bag? ---

b. What is the probability of drawing a blue chip? ---

c. What is the probability of drawing a green chip? ---

d. What is the probability of drawing a black chip? ---

4. A square is chosen randomly from each of the game boards shown. Find the probability that a shaded square is chosen. Give each answer as a fraction, a decimal, and a percent.

a. 

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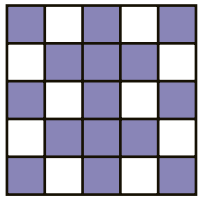
b. 

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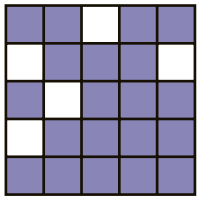
c. 

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5. Liz has several colored ribbons in a drawer. The numbers of ribbons of each color are shown in the table. Liz pulls a ribbon out of the drawer without looking. What are the probabilities that the ribbon picked out of the drawer will be the following colors? Give your answers as fractions.

Color	White	Gold	Black	Green
Number	4	6	7	3

a. White ---

b. Gold ---

c. Black ---

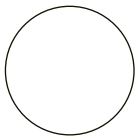
d. Green ---

6. A bowl contains 3 red chips, 4 green chips, and 5 yellow chips.
What is the probability of drawing a red chip on the first draw if a chip is selected at random?

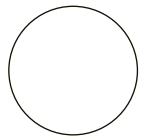
---

7. Complete each of the following spinners so that the event described has the probability given.

a. Spinning an odd number, probability $\frac{2}{3}$.



b. Spinning a prime number, probability $\frac{2}{5}$.



8. Kayne has three bags containing different colored blocks. Calculate the number of blocks in each bag.

a. Bag A: contains 8 green blocks, and the probability of picking a green block is 10%.

---

b. Bag B: contains 8 yellow blocks, and the probability of picking a yellow block is 25%.

---

c. Bag C: contains 8 purple blocks, and the probability of picking a purple block is 50%.

---

9. Given the set A = {5, 10, 15, 20, 25, 30, 35, 40} and the set B = {10, 20, 30, 40, 50, 60},
what is the probability of randomly drawing a number from each set, and getting two multiples of 20?

Name _____ Class _____ Date _____

Lesson 6.2.3

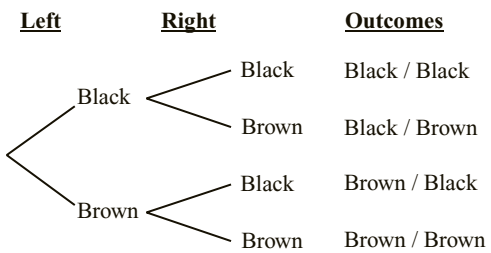
Counting Outcomes

California Standards: Statistics, Data Analysis, and Probability 3.1, 3.3

Example

Joseph has a brown pair of shoes and a black pair of shoes.
His shoe cupboard is dark, so he grabs one left shoe and one right shoe without seeing their color.
a. What is the probability that he grabs the pair of black shoes?
b. What is the probability that he grabs the pair of brown shoes?
c. What is the probability that he grabs a brown shoe and a black shoe?

Solution



Draw a tree diagram to show all the possible outcomes

The tree diagram shows there are 4 possible outcomes

a. P(black pair) = $\frac{1}{4}$ **One of the four possible outcomes shows the pair of black shoes**

b. P(brown pair) = $\frac{1}{4}$ **One of the four possible outcomes shows the pair of brown shoes**

c. P(1 black, 1 brown) = $\frac{2}{4} = \frac{1}{2}$ **Two outcomes show one black and one brown shoe**

1. At Macauley's school, students are allowed to wear blue or black pants, and a red, white, or blue shirt.
a. How many different outfits is Macauley allowed to wear to school? Explain your answer.

---

b. Macauley has one pair of pants of each color, and one shirt of each color. He randomly chooses what to wear each day. What is P(black pants) — the probability that he wears black pants?

---

c. Determine P(red shirt). ---

d. Determine P(grey pants with white shirt). ---

2. The deli has berry, apple, and cherry pies. These can be served plain, or with custard or ice cream. A customer picks a dessert at random.

a. Complete the table to show all the possible desserts.

	Berry (B)	Apple (A)	Cherry (Ch)
Plain (P)			
Ice cream (I)			
Custard (C)			

b. Determine P(berry pie).

---

c. Determine P(custard).

---

d. Determine P(apple pie with ice cream).

---

3. A bag contains 1 red chip, 1 blue chip, and 1 green chip. Darlene draws a random chip from the bag, records the color, and puts it back, before drawing a second chip.
Determine the probability of the following events.

a. P(at least one chip drawn is red) _____ **b.** P(at least one chip drawn is blue) _____

c. P(both chips are the same color) _____ **d.** P(exactly one chip drawn is red) _____

4. Bob has a spinner with 5 equal sections. He spins the spinner twice.
a. How many possible outcomes are there?

_____

b. How many outcomes match the event "the spinner lands on the same section twice"?

_____

5. Maple calculates that there are 36 possible outcomes when spinning her spinner twice.
How many sections does the spinner have? Explain your answer.

_____

6. Roberta has a spinner with 7 sections. She says there are 49 possible outcomes from spinning her spinner twice, and 56 possible outcomes from spinning it three times. Explain whether she is right.

_____

7. Devin tosses a coin, then rolls a normal 6-sided die.
a. Draw a tree diagram to show all the possible outcomes.

b. What is the probability that he rolls a 6? _____

c. What is the probability that the coin will land heads up? _____

8. Brett has 3 different-colored cups and three different balls. Brett randomly puts one ball in one of the cups.
a. Complete the grid to show all the possible outcomes.

b. What is the probability that the red cup contains the tennis ball?

_____

		Ball		
		Tennis	Golf	Squash
Cup	Red			
	Blue			
	Orange			

9. Linda has 6 different-colored buttons on a tray: red, green, blue, yellow, orange, and purple.
She picks a button without looking, records its color, and replaces it, before picking another button.

a. How many possible outcomes are there? _____

b. Determine P(green on the first pick). _____

c. Determine P (green on both picks). _____

Name _____ Class _____ Date _____

Lesson 6.2.4 Probability of an Event Not Happening

California Standard: Statistics, Data Analysis, and Probability 3.3

Example

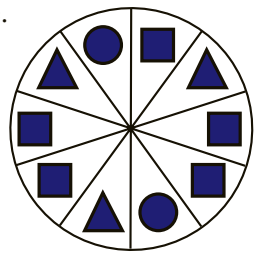
Sam spins a spinner with 9 equal sections, numbered 1 through 9.
a. What is the probability of rolling a number less than 5?
b. What is the probability of not rolling a number less than 5?

Solution

a. $\frac{4}{9}$ **4 out of 9 outcomes are less than 5**

b. $\frac{5}{9}$ **5 out of 9 outcomes are not less than 5 (5, 6, 7, 8, and 9)**

1. Find the following probabilities for the result of spinning the spinner shown, once.



a. P(square) _____ **b.** P(not square) _____

c. P(circle) _____ **d.** P(not circle) _____

e. P(triangle) _____ **f.** P(not triangle) _____

Example

There is a 20% chance that Sabrina will run the mile race tomorrow.
What is the probability that Sabrina will not run the race?

Solution

Sabrina must either run the race or not run the race, so P(she runs) + P(she doesn't run) = 100%.
So 20% + P(she doesn't run) = 100
P(she doesn't run) = 100% – 20% = 80%

2. What is the probability of each of the following?

a. Not drawing a blue chip, where P(drawing a blue chip) = $\frac{3}{8}$. _____

b. Not seeing Fred tomorrow, where P(seeing Fred tomorrow) = $\frac{2}{5}$. _____

c. Not going to the movies this evening, where P(going to the movies this evening) = $\frac{3}{4}$.

_____

d. Not raining tomorrow, where P(rain tomorrow) = 30%. _____

3. Ana took her broken bicycle to the repair shop to have the frame straightened. The repairman told her there was a 30% chance that he wouldn't be able to fix it. What is the probability that he will be able to fix it?

---

4. Edgar is at his friend's house playing pool. There is a 1 in 4 chance that he will make his next shot. What is the probability that he will miss the shot?

---

5. What is the probability that the power will go off during a storm, if the probability of it staying on is 78%?

---

6. Sandra has a $\frac{1}{3}$ chance of making the team.

What is the probability that Sandra will not make the team?

---

7. Colin has 15 cubes in a box. Four of the cubes are red, five are yellow, and six are blue. Determine the probability of the following events.

a. P(picking yellow) ---

b. P(not picking yellow) ---

c. P(not picking red) ---

d. P(not picking blue) ---

Name _____ Class _____ Date _____

Lesson 6.2.5

Venn Diagrams

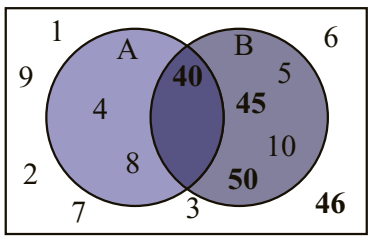
California Standard: Statistics, Data Analysis, and Probability 3.1

Example

Fifty counters, numbered 1–50, are in a bag. One counter is selected at random. Tamara is making a Venn diagram to decide the probabilities of the following events:
Event A: The counter shows a multiple of 4.
Event B: The counter shows a multiple of 5.
Place each of the numbers below on the diagram.

a. 40 **b.** 45 **c.** 46 **d.** 50

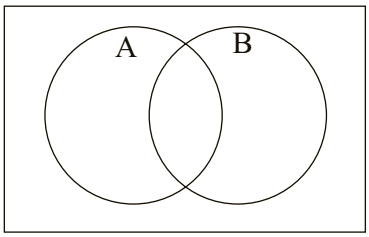
Solution



40 is a multiple of 4 and 5, so goes in both circles
45 is a multiple of 5, but not of 4, so goes in circle B
46 is not a multiple of 4 or 5, so goes outside the circles
50 is a multiple of 5, but not of 4, so goes in circle B

1. A 20-section spinner is labeled with the numbers 1–20. It is spun once.
Draw a Venn diagram to show which outcomes match the following events.

a. Event A: "number is odd"
Event B: "number is a multiple of 9"

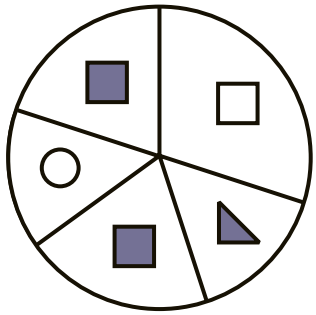


b. Event A: "number is a multiple of 10"
Event B: "number is a multiple of 5"

c. Event A: "number has an even ones digit"
Event B: "number has an even tens digit"

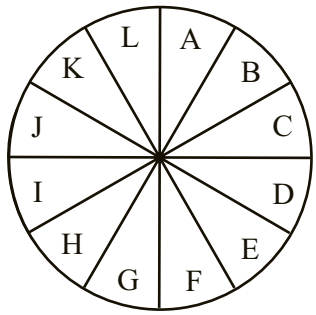
d. Event A: "number has a ones digit equal to 5"
Event B: "number is even"

2. Ewan spins the spinner in the diagram. Use a Venn diagram to show the outcomes that match the following events.
Event A: "spinning a shaded shape"
Event B: "spinning a square"

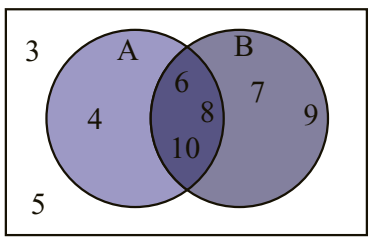


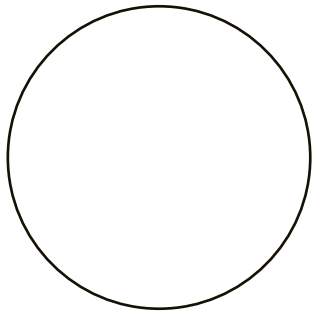
3. Kelly spins the spinner in the diagram. Use a Venn diagram to show the outcomes that match the following events.
Event A: "spinning a vowel"
Event B: "spinning a consonant"

Vowels are the letters A, E, I, O, and U.
All the other letters are consonants.



4. Mario drew the Venn diagram below to show how many outcomes match the following events:
Event A: "spinning an even number" Event B: "spinning a number greater than 5"
Draw one possible spinner that Mario could have used.

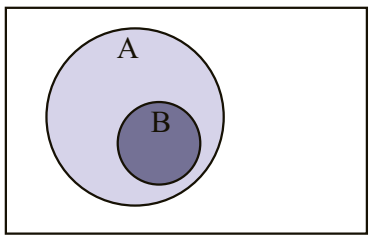




5. Lyla said that the circles in a Venn diagram must always overlap. Is Lyla's statement true? Explain your answer.

---

6. The numbers 1–200 are written on pieces of paper, and then put into a bag. One piece of paper is selected at random. Jacques drew the Venn diagram shown on the left to show which numbers match two events.
Event A is "number selected is less than 100."
Suggest what event B could be.



Name _____ Class _____ Date _____

Lesson 6.2.6

Combining Events

California Standards: Statistics, Data Analysis, and Probability 3.3, 3.4

Example

Given the set of numbers 1 through 12, find the probability of each of the following events, if one number is selected at random.

Answer parts a–d using the formula $\text{P(event)} = \dfrac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

a. P(drawing an odd number)
b. P(drawing a multiple of 3)
c. P(drawing an odd multiple of 3)
d. P(drawing a number that is odd or a multiple of 3)
e. Show how you could have answered part d using your answers to parts a–c.

Solution

a. $\frac{6}{12} = \frac{1}{2}$ **6 out of the 12 numbers are odd (1, 3, 5, 7, 9, and 11)**

b. $\frac{4}{12} = \frac{1}{3}$ **4 out of the 12 numbers are multiples of 3 (3, 6, 9, and 12)**

c. $\frac{2}{12} = \frac{1}{6}$ **Of the 6 odd numbers, 2 are also multiples of 3 (3 and 9), so 2 out of the 12 numbers are odd multiples of 3**

d. $\frac{8}{12} = \frac{2}{3}$ **8 out of the 12 numbers are odd or multiples of 3**

e. $\text{P(A or B)} = \text{P(A)} + \text{P(B)} - \text{P(A and B)} = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3+2-1}{6} = \frac{4}{6} = \frac{2}{3}$

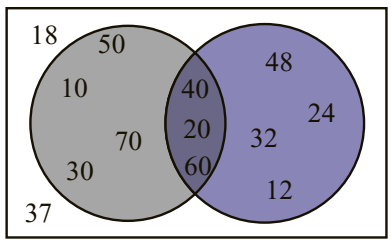
1. Determine the probabilities below, if a single number is drawn from those on the Venn diagram shown.

a. P(multiple of 4) _____

b. P(multiple of 10) _____

c. P(multiple of 4 and a multiple of 10) _____

d. P(multiple of 4 or a multiple of 10) _____



2. Use probability notation to complete the following statement.

P(A or B) = P(A) + P(B) – _____

3. One counter is selected from a set of 15, which are numbered 1 through 15. Find the following:

a. P(drawing a prime number) _____

b. P(drawing an odd number) _____

c. P(drawing an odd prime number) _____

d. Use your answers to parts a–c to find P(drawing a number that is odd or prime) _____

4. For which of the following pairs of events is P(A and B) equal to zero?

a. When rolling a regular, six-sided die:
Event A — "rolling a number greater than 5" Event B — "rolling a prime number"

_____

b. When selecting an integer from 1 to 20:
Event A — "selecting an odd number" Event B — "selecting a multiple of 3"

_____

c. When selecting an integer from 1 to 10.
Event A — "selecting a number less than 4" Event B — "selecting 1"

_____

d. When selecting an integer from 1 to 15.
Event A — "selecting a number greater than 10" Event B — "selecting a number less than 10"

_____

5. A bag contains 12 chips, numbered 1 through 12. The odd-numbered chips are colored blue, and the even-numbered chips are colored red. A chip is drawn from the bag. Find the following probabilities.

a. P(red) _____ **b.** P(less than 5) _____

c. P(red and less than 5) _____ **d.** P(red or less than 5) _____

6. When drawing one number from the set of integers from –4 to 4 inclusive, determine the following probabilities.

a. P(greater than –2) _____ **b.** P(negative number) _____

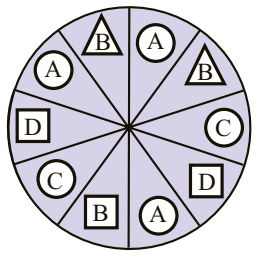
c. P(negative number greater than –2) _____

d. P(greater than –2 or negative) _____

7. Rachel spins the spinner shown. Determine the following probabilities. Write your answers as decimals.

a. P(square) _____ **b.** P(B) _____

c. P(B and square) _____ **d.** P(B or square) _____



8. When picking a number at random from the integers 1–100, what is the probability of picking either a multiple of 7 or a multiple of 15? Use the formula P(A or B) = P(A) + P(B) – P(A and B)

_____

_____

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Lesson 6.3.1

Independent and Dependent Events

California Standard: Statistics, Data Analysis, and Probability 3.5

Example

Mrs. Davey is making lunch for her sons Jim and Tom. She has enough salami for one sandwich. Are the events "Jim gets salami for lunch" and "Tom gets salami for lunch" dependent? Explain your answer.

Solution

Yes, they are dependent. If Jim gets salami for lunch, Tom cannot because there is only enough for one sandwich. Equally, if Tom gets the salami, Jim cannot also have it.

The probability of one event happening affects the probability of the other

1. James' cookie jar contains oatmeal cookies and chocolate-chip cookies. He takes two cookies from the jar, one at a time, without looking, then eats them. Are the events "the first cookie is oatmeal" and "the second cookie is chocolate-chip" dependent? Explain your answer.

---

2. Every day, Mrs. Blest chooses the homeroom monitor by picking one of the student's names from a bowl. She replaces the name, so all names are in the bowl for the next morning. Is a student's chance of being picked each day dependent on who was monitor the previous day? Explain your answer.

---

3. Kelly-Anne has a spinner numbered 0 through 9. She spins the spinner twice, using the first outcome as the tens digit of a two-digit number, and the second outcome as the ones digit. Are the events "the ones digit is odd" and "the tens digit is even" dependent on one another? Explain your answer.

---

4. Lorenzo has rolled a normal 6-sided die three times. He rolled 4 each time.
Is the fourth roll of the die independent of the previous three rolls? Explain your answer.

---

5. A bowl contains tiles marked with the letters of the alphabet. Tracey draws one tile, and doesn't replace it. Tyler then draws one tile. Are the events "Tracey draws the letter R" and "Tyler draws the letter A" dependent or independent? Explain your answer.

---

6. A bowl contains a number of colored balls. Manuel draws a ball, replaces it, then draws another.
Are the events "the first ball drawn is green" and "the second ball drawn is yellow" dependent?
Explain your answer.

---

7. State whether each of the following pairs of events is independent or dependent.

a. You draw a chip from a bag, replace it, and draw again.

---

b. You draw a chip from a bag, do not replace it, and draw again.

---

c. You take a piece of candy from a bowl and eat it. You then take another piece of candy.

---

8. Ronnie has a box of cards bearing the letters A, B, C, D, and E.
Ronnie draws one card, then Sadia draws one card.

a. Describe how the two cards could be drawn so that the outcomes of each draw are independent.

---

b. Describe how the two cards could be drawn so that the outcome of Sadia's draw is dependent on the outcome of Ronnie's draw.

Name _____ Class _____ Date _____

Lesson 6.3.2

Events and Probabilities

California Standard: Statistics, Data Analysis, and Probability 3.5

Example

Damon has a regular 52-card deck, where half the cards are red and half are black. Damon picks a card, replaces it, and picks another. Show that the events "the first card is red" and "the second card is black" are independent.

Solution

Suppose Damon picks a red card first. The probability of picking a black card second is $\frac{26}{52} = \frac{1}{2}$.

Suppose Damon picks a black card first. The probability of picking a black card second is $\frac{26}{52} = \frac{1}{2}$.

The probability of the second event is the same whether the first event happens or not, so the events are independent.

1. There are 6 sweets in a bag, of which 3 are strawberry flavor and 3 are banana flavor. Kasey takes a sweet from the bag, eats it, then offers the bag to her friend Tryan.

a. If Kasey gets a strawberry sweet, what is the probability of Tryan also getting a strawberry sweet?

---

b. If Kasey gets a banana sweet, what is the probability of Tryan getting a strawberry sweet?

---

c. Are the events "Kasey gets a strawberry sweet" and "Tryan gets a strawberry sweet" dependent? Explain your answer.

---

2. There are 3 cherry popsicles and 2 raspberry popsicles in a bag. Robert and Ann both want a popsicle. Ann chooses one and then Robert chooses one. Show that the events "Ann's popsicle is cherry" and "Robert's popsicle is raspberry" are dependent.

---

3. Tyrese has 7 hip-hop CDs and 5 drum-and-bass CDs in a box. He picks a CD, listens to it, puts it back in the box, then picks another CD without looking. Show that the event "the second CD Tyrese picks is drum and bass" is independent of the event "the first CD Tyrese picks is hip-hop."

---

4. Juwan has 6 cards numbered 10, 12, 20, 24, 32, and 40 in a box. He is going to pick a card, record the number, put the card to one side, then pick another card. Will his two selections be dependent or independent? Explain your answer.

---

5. Grace has a pack of star-shaped stickers, containing 3 silver, 2 gold, and 5 bronze stickers. Grace wants a gold sticker, and wants to increase the likelihood of picking one out with each random pick she makes. If she picks a sticker that is not gold, should she replace it or keep it out when she picks another? Explain your answer in terms of probability.

---

6. Marvin has a bag containing 5 blocks, labeled A to E. He is conducting probability experiments involving the following events.
Event A: He picks the letter A from the bag with his first pick.
Event B: He picks the letter A from the bag with his second pick.

a. In his first experiment, he replaces the first block before making his second pick.
Calculate P(B) twice — once if event A happens, and once if event A does not happen.
Are events A and B independent? Explain your answer.

---

b. In his second experiment, he does not replace the first block before making his second pick.
Calculate P(B) twice — once if event A happens, and once if event A does not happen.
Are events A and B independent? Explain your answer.

---

7. Josie has a spinner with 6 equal-sized sections, colored blue, pink, green, red, yellow, and white. She spins it twice. By considering probability, determine whether each of the following pairs of events are dependent or independent. Explain your answer.

a. Event A: Josie spins blue on the first spin.
Event B: Josie spins pink on the second spin.

---

b. Event A: Josie spins green on the first spin.
Event B: Josie spins green on the second spin.

---

8. Mayra has a bag containing tiles labeled with the letters A, B, E, G, I, M, O, R, and U. She draws two tiles in turn. If the first letter is a vowel, she keeps it. If it is a consonant, she puts it back. Determine whether the following pairs of events are dependent or independent.

a. Event A: Mayra draws a vowel with her first pick
Event B: Mayra draws a consonant with her second pick

---

b. Event A: Mayra draws a consonant with her first pick
Event B: Mayra draws a vowel with her second pick

Name _____ Class _____ Date _____

Lesson 6.3.3

Calculating Probabilities of Independent Events

California Standard: Statistics, Data Analysis, and Probability 3.4

Example

Andre has a bowl containing 6 red chips and 4 blue chips.
He draws one chip, notes its color, and replaces it, before drawing a second chip.
What is the probability of Andre drawing a blue chip followed by a red chip?

Solution

The first chip is replaced, so the events are independent. This means you can use the formula P(A and B) = P(A) × P(B)

$P(\text{blue}) = \frac{4}{10} = \frac{2}{5}$

$P(\text{red}) = \frac{6}{10} = \frac{3}{5}$

Calculate the probability of each event happening separately

$P(\text{blue, then red}) = P(\text{blue}) \times P(\text{red}) = \frac{2}{5} \times \frac{3}{5} = \mathbf{\frac{6}{25}}$

Multiply the probabilities to find the probability of both events happening

1. Emma has a bowl containing tiles numbered 1, 1, 1, 2, 2, 3, 4, 5, 5, and 6. She draws three tiles to form the digits of a 3-digit number in the order the tiles were drawn. After drawing each tile, she replaces it before drawing the next one. Calculate the probability of drawing:

a. a 1 on the first draw. _____

b. a 2 on the second draw. _____

c. a 3 on the third draw. _____

d. Are these events dependent or independent? _____

e. Calculate the probability of the tiles forming the number 123. _____

2. Ava has a bag containing 6 red blocks, 8 blue blocks, and 4 green blocks. She draws one block, records the color, and returns the block to the bag, before drawing a second block.

a. Are the events in this situation dependent or independent?

_____

Calculate the probability of each of the following events.

b. Drawing a red block followed by a blue block.

_____

c. Drawing a red block followed by a green block.

_____

d. Drawing a blue block followed by a red block.

_____

e. Drawing two green blocks.

_____

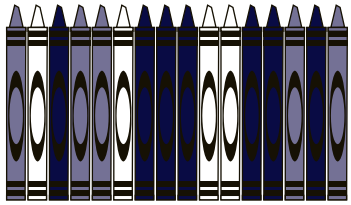
3. Horace placed the crayons shown into a box. He then picked one out without looking, noted the color, and then replaced it, before picking another crayon out. Find the probability of each of the following events.

a. P(dark blue then white) ----------

b. P(light blue then white) ----------

c. P(white then light blue) ----------

d. P(dark blue twice) ----------



4. Vonda has a biased number cube. Unlike a normal number cube, the probabilities of the various outcomes are different. The rolls are still independent, however.

The probability of the number cube landing on 1 is $\frac{1}{6}$.

a. If the probability of the cube landing first on 1, and then on 2 is $\frac{1}{18}$, what is the probability of rolling a 2?

---

b. If the probability of the cube landing first on 3, and then on 1 is $\frac{1}{12}$, what is the probability of rolling a 3?

---

5. Webster rolls two normal 6-sided dice. One die is blue, the other is red. Calculate the probability of the following events.

a. The blue die lands on a number less than 3, and the red die lands on a number greater than 3.

---

b. The two numbers sum to give 6 and the blue die is less than the red die.

---

6. Miriam has 10 nickels, 6 dimes, and 4 quarters in her purse. She pulls out three coins, one at a time. Miriam says the probability of the total being more than \$0.60 is the same whether she replaces the coins after each pick or not.

a. Is Miriam correct? Explain your answer.

---

b. What is the probability of the total being more than \$0.60 if she replaces each coin after picking it?

Name _____ Class _____ Date _____

Lesson 6.4.1

Relative Frequency

California Standard: Statistics, Data Analysis, and Probability 3.2

Example

Over his last 320 at bats for his baseball team, Armando has hit 122 balls.
Estimate the probability that he will hit the next ball he faces.

Solution

Use the relative frequency of Armando's hits as a measure of probability.
Relative frequency = 122 ÷ 320 = **0.38125**

1. Craig has scored a home run on 23 of his last 169 at bats.
What is the relative frequency of his home runs?

---

2. Professor Parker is carrying out an experiment. For each trial, there are four possible outcomes, A, B, C, or D. The results of Professor Parker's first 2000 trials are shown below.
Complete the table by calculating the relative frequency of each event.

Event	A	B	C	D
Frequency	764	327	643	266
Relative frequency	----------	----------	----------	----------

3. A company tests a sample of the parts they manufacture each day for faults. If the relative frequency of faulty parts is greater than 0.005, all parts produced that day are recalled.
The table below shows the data collected over one week. On which days were parts recalled?

Day	Parts Checked	Faulty Parts
Monday	1000	3
Tuesday	1000	8
Wednesday	2000	8
Thursday	2000	15
Friday	5000	20

---

4. Joe flips a fair coin 100 times, and it comes up heads 58 times.

a. What is the experimental probability of getting a head?

---

b. What is the theoretical probability of getting a head?

---

5. So far this season, Janice has made 21 free throws out of 35 attempts.

a. What is the relative frequency of her making free throws?

---

b. Estimate the probability of Janice making her next free throw.

---

6. A company makes light switches. They only ship the switches if the relative frequency of defective switches is less than 0.05. In one batch, 800 switches were checked, and 33 were found to be defective. Should the company ship this batch?

---

7. A spinner with three equal sections lands on a blue section 64 times out of 100 spins.

a. What is the experimental probability of landing on blue?

---

b. Two of the spinner's sections are blue. What is the theoretical probability of spinning blue?

---

c. How does the experimental probability compare with the theoretical probability? How would you expect the experimental probability to change if the spinner was spun 10,000 times?

Name _____ Class _____ Date _____

Lesson 6.4.2

Making Predictions

California Standard: Statistics, Data Analysis, and Probability 3.2

Example

Darius is practicing magic tricks for a show he will perform. 15 out of the last 34 tricks he has tried have worked. If Darius performs 12 tricks in the show, estimate how many will work.

Solution

Calculate the experimental probability of a trick working:

$15 \div 34 = 0.44$

Multiply the probability by the number of tricks he will perform:

$0.44 \times 12 = 5.29$

Make sure the answer is a reasonable value for the situation

Approximately 5 or 6 of the tricks Darius performs are likely to work.

1. A clothing company checked a sample of 200 shirts, and found that 15 were below the required standard. If the company produces 3000 shirts a day, estimate how many will be below standard each day.

---

2. Carolina sells products over the phone. Yesterday, she called 100 people, and made 5 sales. If she calls 1500 people next month, estimate how many sales she will make.

---

3. John is a car salesman. He estimates that for every 12 customers he approaches, he makes one sale. Estimate how many customers John will need to approach to make 15 sales.

---

4. Jason is concerned about the risk of harm to motorists who don't wear their seat belt. He stood on a local street corner, and counted 11 out of 100 drivers not wearing their seat belt. If there are 5600 drivers in Jason's neighborhood, estimate how many drive without a seat belt.

---

5. Rex's batting average was .300 in April, .297 in May and .305 in June.
If Rex gets 90 at bats in July, how many hits would you predict he would make?

---

6. Delton, a bicycle enthusiast, was concerned when he saw some children riding their bicycles without wearing helmets. He watched, and saw that 3 out of the 20 children were not wearing helmets. Delton estimates there are 2000 children with bicycles in his neighborhood. Estimate how many children in Delton's neighborhood cycle without wearing a helmet.

---

7. Mr. O'Neil works in a coffee shop. One morning, he noted that 8 out of 20 customers did not put sugar in their coffee. The shop serves approximately 300 cups of coffee every week.
Estimate how many customers do not put sugar in their coffee each week.

---

8. Josephine often runs promotions on new products in her store. In the last candy-bar promotion she ran, 2 out of every 10 people that bought candy chose the new candy bar. Josephine is planning a new candy-bar promotion. She estimates that 450 customers will come to the store to buy candy during this promotion. How many candy bars should she order for the promotion?

---

9. Boris and his team test tennis balls to make sure that they have the right pressure.
One day, 45 out of the first 300 balls they check have the wrong pressure, and have to be recycled.
During the day, the factory they work in produces a total of 4000 tennis balls.
How many of these would you estimate will need to be recycled?

---

10. Mr. Smith needs to know how many copies of a magazine to order to keep his store stocked.
The salesman gave him 20 copies, and they were bought by the first 56 customers. Mr. Smith estimates that he gets 900 customers a week. How many of this week's magazine should he buy?

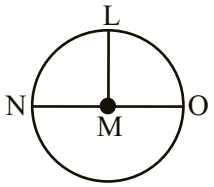
Name _____ Class _____ Date _____

Lesson 7.1.1

Parts of a Circle

California Standards: Algebra and Functions 3.1, 3.2

1. Which distance in the diagram below is equal to:

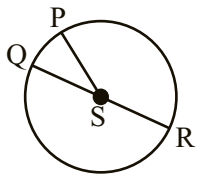


Remember: describe distances using endpoints — so the distance between A and B is written AB.

a. the diameter of the circle? _____

b. the radius of the circle? _____

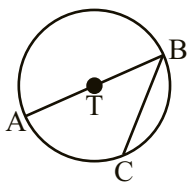
2. Which distance in the diagram below is equal to:



a. the radius of the circle? _____

b. the diameter of the circle? _____

3. In this circle:



a. is AB the diameter of the circle? Explain your answer.

_____

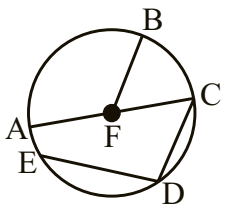
b. is BC the radius of the circle? Explain your answer.

_____

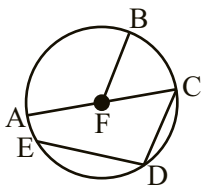
4. Tick the correct label for the radius of the circle shown.

☐ ED ☐ AC

☐ FB ☐ CD



5. Which of the following is a diameter of the circle shown?



☐ ED ☐ AC

☐ FB ☐ CD

6. Decide whether each of the following distances is a diameter or not a diameter.

a. The distance around the top of a glass. ----------

b. The distance across a circular clock face from 1 to 8. ----------

c. The distance from one end of a football to the other. ----------

d. The distance across a circular clock face from 4 to 10. ----------

7. Between how many pairs of points can you measure a circle's diameter? Explain your answer.

---

8. Greg draws two circles, circle A and circle B. The diameter of circle A is three times that of circle B. Complete the following sentence:

The diameter of circle A is _____ times the radius of circle B.

9. If r is the radius of a circle and d is its diameter, find each of the missing measures below.

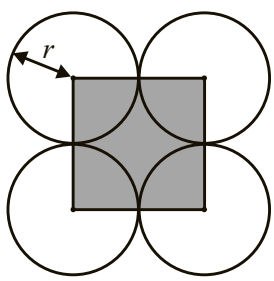
a. If $r = 18$ cm, then $d =$ ---------- **b.** If $r = 27$ in., then $d =$ ----------

c. If $d = 38$ ft, then $r =$ ---------- **d.** If $d = 0.5$ cm, then $r =$ ----------

10. Hania is making a fabric cover for the top of a circular stool. The stool has a radius of 7 inches, and she wants the cover to hang between 1 and 2 inches over the edge.
Which of the following sizes of material does she need?

☐ 14-inch diameter ☐ 17-inch diameter ☐ 14-inch radius ☐ 17-inch radius

11. The diagram to the right shows four circles of radius r.
What is the area of the shaded square?



Name _____ Class _____ Date _____

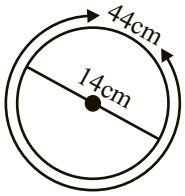
Lesson 7.1.2

Circumference and π

California Standards: Algebra and Functions 3.1, 3.2, Measurement and Geometry 1.1, 1.2

Example

Find the ratio between the circumference and the diameter of the circle shown below.



Solution
44 ÷ 14 = 3.14 rounded to the nearest hundredth.

This answer is always the same for any circle.

1. Which of the following distances is a circumference?

- ☐ The distance around the face of a cube.
- ☐ The distance around a text book.
- ☐ The distance around a nickel.
- ☐ The distance around one of the Hawaiian islands.

2. Estimate the circumference of each circle with the given radius or diameter. Use 3.14 as the ratio of the circumference to the diameter.

a. diameter = 10 inches. _____ **b.** diameter = 14 cm. _____

c. diameter = 8 ft. _____ **d.** radius = 6 yds. _____

Example

Given the diameter of a circle is 10 cm, find its circumference. Use $\pi \approx 3.14$.

Solution
$C = \pi d$.
$C \approx 3.14 \times 10 = \mathbf{31.4\ cm}$.

3. Use $\pi \approx \frac{22}{7}$ to approximate the circumference of a circle whose diameter is 28 inches.

_____

4. Use $\pi \approx 3.14$ to approximate the circumference of a circle whose radius is 5 cm.

_____

5. Using $\pi \approx 3.14$, approximate the circumference of circles with the following measurements:

a. $d = 20$ in ----------

b. $d = 100$ cm ----------

c. $r = 500$ cm ----------

d. $r = 25$ ft ----------

6. Using $\pi \approx 3.14$, approximate the circumference of circles with the following measurements:

a. $d = 15$ in ----------

b. $d = 35$ cm ----------

c. $r = 45$ cm ----------

d. $r = \frac{1}{2}$ mi ----------

7. Charles needs a new tire for his wheelbarrow. He knows the circumference of the tire is 56.52 inches, but he needs to know its diameter so he can buy the correct size of tire at the store. Approximately what diameter of tire does Charles need to buy? (Use $\pi \approx 3.14$)

---

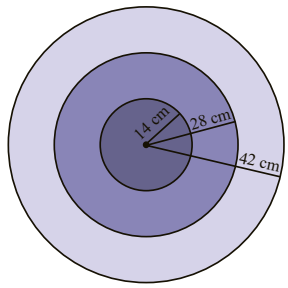
8. Jack has a circular table cloth with a radius of 12 inches.
Use $\pi \approx 3.14$ to approximate the circumference of the tablecloth.

---

9. Sandra's tennis ball can is the height of exactly 3.1 tennis balls.
Is the circumference of Sandra's can larger or smaller than the height of the can? Explain your answer.

---

10. Use $\pi \approx \frac{22}{7}$ to approximate the circumference of each of the circles shown in the target on the right.



11. Alex said, "When the diameter of a circle is doubled, the circumference is doubled."
Show that Alex is correct.

---

12. $\pi = 3.14159265...$ can be approximated by 3.14 and by $\frac{22}{7}$.

a. Which approximation is larger? ----------

b. Which approximation is closest to the true value of π? ----------

Name _____ Class _____ Date _____

Lesson 7.1.3

Area of a Circle

CA Standards covered: Measurement and Geometry 1.1, 1.2

Example

Find the area of a circle with a radius of 10 in.

Solution

$A = \pi r^2 \approx 3.14 \times 10 \times 10 =$ **314 sq in**.

1. Which of the following is the correct representation for the area of a circle with diameter 40 cm.

☐ $A = 40\pi$ cm^2 ☐ $A = \left(\frac{40}{2}\right)\pi$ cm^2

☐ $A = 40^2\pi$ cm^2 ☐ $A = \left(\frac{40}{2}\right)^2\pi$ cm^2

2. Determine the area of a circle whose radius is 15 feet. Use $\pi = 3.14$.

---

3. Determine the area of a circle whose diameter is 14 inches. Use $\pi = 22/7$.

---

4. Using $\pi \approx 3.14$, find the area of each circle with the given radius or diameter.

a. $r = 20$ cm _____

b. $r = 4$ in. _____

c. $d = 30$ in. _____

d. $d = 6$ ft _____

5. Using $\pi \approx \frac{22}{7}$, find the area of each circle with the given radius or diameter.

a. $r = 7$ in. _____

b. $r = 21$ cm _____

c. $d = 56$ cm _____

d. $d = 84$ in. _____

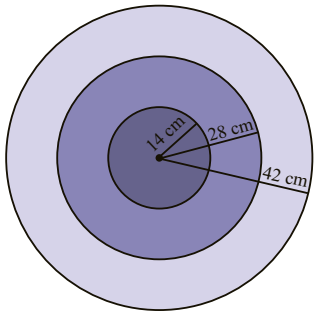
6. For the target on the right, find the area of the inside circle, then the area of each ring.

Use $\pi \approx \frac{22}{7}$.

Inside circle: ----------

1st ring: ----------

2nd ring: ----------



7. The pizza place nearest to Jose's house sells pizzas with a diameter of 9-inches or 13-inches. Using $\pi = 3.14$, estimate which would give Jose the greatest area of pizza: two 9 inch pizzas or one 13 inch pizza.

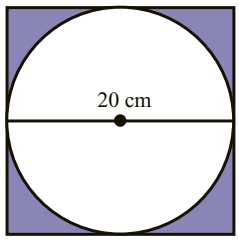
---

8. The local park has six circular flower beds, each with a diameter of 50 ft. If one sack of fertilizer covers 5000 ft^2, estimate how many sacks of fertilizer the park needs to buy to fertilize all the flower beds once. Use $\pi = 3.14$.

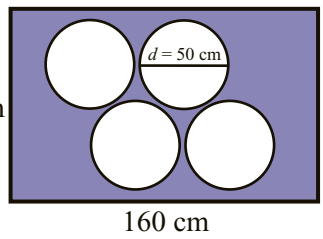
---

9. Given that the diameter of the circle is 20 cm, estimate the area of the shaded part of the square shown. Use $\pi = 3.14$.

Remember that the formula for the area of a square is: $A = s^2$.



10. Olivia is making circular pillows for her sofa. She has bought a piece of fabric 130 cm × 160 cm, and cut out 4 circles, each with a diameter of 50 cm. Estimate the area of fabric left after the circles have been cut out. Use $\pi = 3.14$.



Name _____ Class _____ Date _____

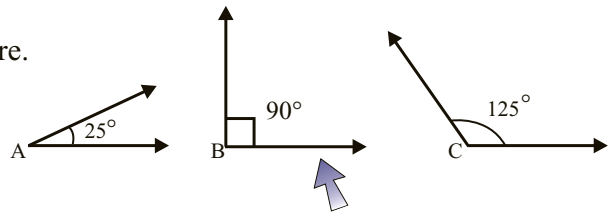
Lesson 7.2.1

Describing Angles

California Standards: Measurement and Geometry 2.1, 2.2

Example

Classify each angle below using its measure.



Solution

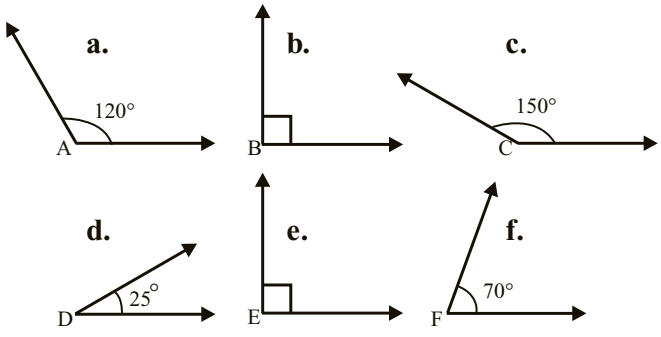
A is acute because its measure is between 0° and 90°.

B is right because its measure is exactly 90°.

C is obtuse because its measure is between 90° and 180°.

The box drawn in the angle shows you that it is a right angle, and measures exactly 90°.

1. Determine whether each of the following angles is acute, obtuse, or right.

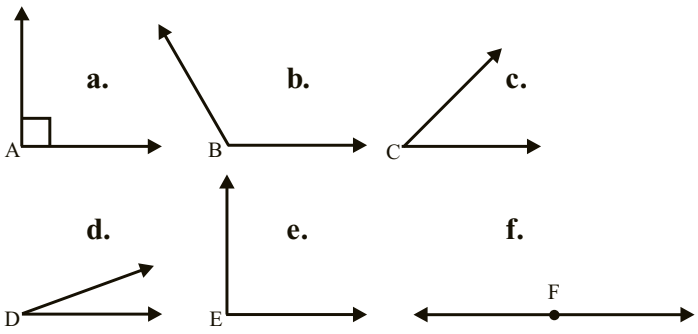


a. _____ **b.** _____

c. _____ **d.** _____

e. _____ **f.** _____

2. Decide whether each of the following angles is acute, obtuse, right, or straight.

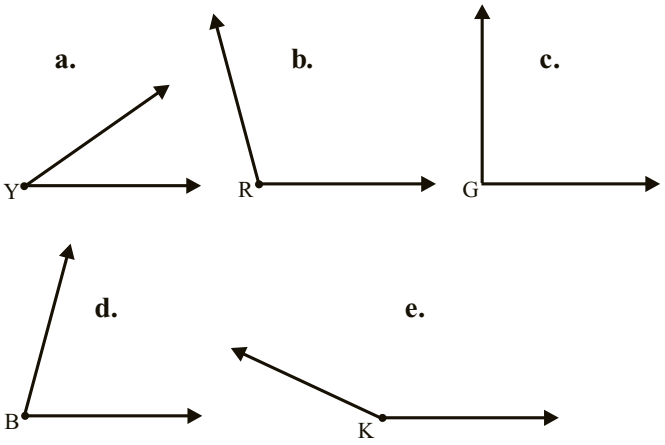


a. _____ **b.** _____

c. _____ **d.** _____

e. _____ **f.** _____

3. Measure each of the following angles with a protractor, and decide whether it is acute, obtuse, or right.



a. _____° _____

b. _____° _____

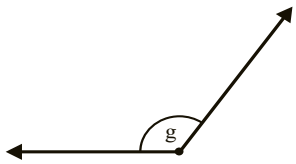
c. _____° _____

d. _____° _____

e. _____° _____

4. Jesse measured angle *g* and said it was 55°.
Lisa said it was much greater. Which student was correct?

_____



5. Using your protractor, draw angles with the following measures, then state whether it is an acute angle, an obtuse angle, or a right angle:

a. 50°

_____

b. 145°

_____

c. 90°

_____

d. 73°

_____

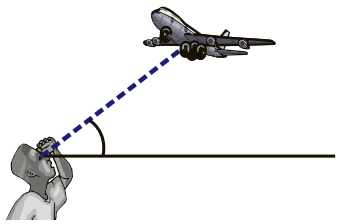
6. Carl is making a sculpture out of iron rods.

He uses three rods, each bent to make a different angle: 40°, 80°, and 120°.

Use your protractor to draw the angle of each iron rod. Label each angle as acute, obtuse, or right.

7. Eric is looking at the airplane.
How large is the angle shown, and what type of angle is it?

_____



Name _____ Class _____ Date _____

Lesson 7.2.2

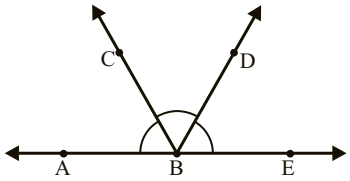
Pairs of Angles

California Standard: Measurement and Geometry 2.1

Example

Determine which pairs of angles are adjacent.

∠ABC is the same as saying "the angle ABC".



Solution

∠ABC and ∠CBD are adjacent
∠CBD and ∠DBE are adjacent
∠ABC and ∠DBE are <u>not</u> adjacent

1. From the diagram, decide which of the named pairs of angles are adjacent.

a. ∠ACB and ∠BCE ----------

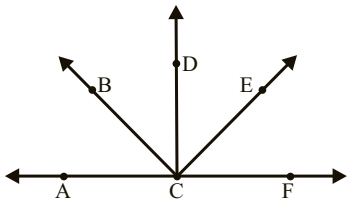
b. ∠BCD and ∠DCF ----------

c. ∠ACD and ∠DCF ----------

d. ∠ACD and ∠FCE ----------

e. ∠ACE and ∠ACB ----------

f. ∠BCD and ∠ECF ----------

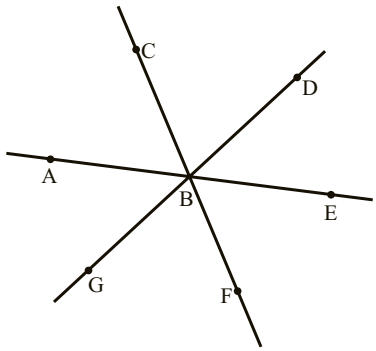


2. The figure shows three lines intersecting at one point. Name six pairs of vertical angles.

---------- ----------

---------- ----------

---------- ----------

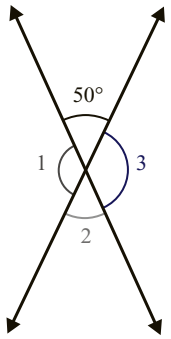


3. Determine the measure of angles 1 – 3 in the diagram shown.

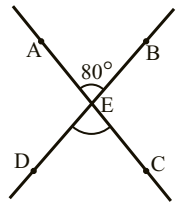
1 = ----------

2 = ----------

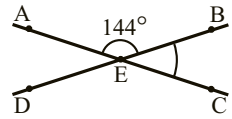
3 = ----------



4. Find ∠DEC.

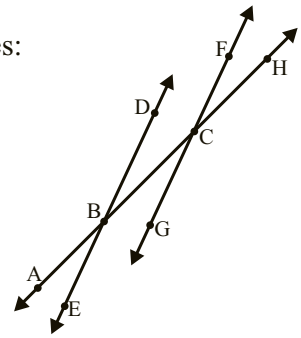


5. Find ∠CEB.



6. Is it possible to have adjacent angles that are also vertical angles? Explain your answer.

7. The diagram shows parallel lines DE and FG crossing the line AH. If ∠ABE = 20°, write down the measures of each of the following angles:



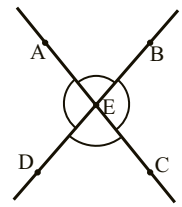
a. ∠ABD _____ **b.** ∠EBC _____

c. ∠DBC _____ **d.** ∠GCB _____

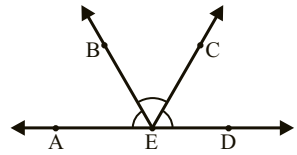
e. ∠BCF _____ **f.** ∠BCH _____

g. ∠FCH _____

8. Write down one pair of vertical angles shown in the diagram.



9. Write down one pair of adjacent angles shown in the diagram.



10. Find the measures of angles a – e in the diagram below.

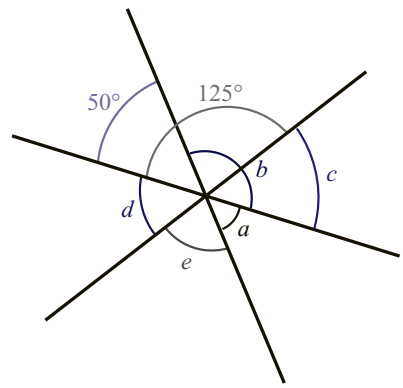
a = _____

b = _____

c = _____

d = _____

e = _____



Name _____ Class _____ Date _____

Lesson 7.2.3

Supplementary Angles

California Standards: Measurement and Geometry 2.1, 2.2

Example

Which of these four angles are supplementary?

$\angle A = 155°, \angle B = 205°, \angle C = 125°, \angle D = 55°$

Supplementary angles are any two angles whose measures sum to 180°.

Solution

$\angle C$ and $\angle D$

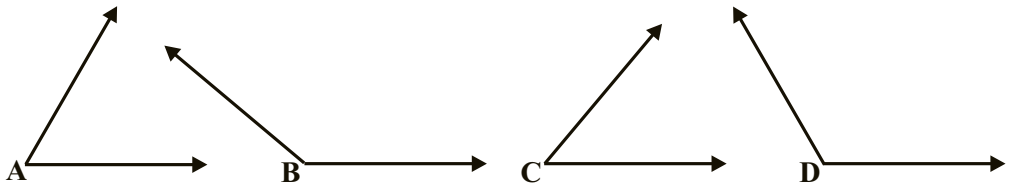
These are the two angles whose measures sum to 180°.

1. Find the supplement for each of the following angles.

a. $\angle A = 45°$ **b.** $\angle B = 90°$ **c.** $\angle C = 125°$ **d.** $\angle D = 6°$

---

2. Measure these angles with a protractor, and find which are supplementary.



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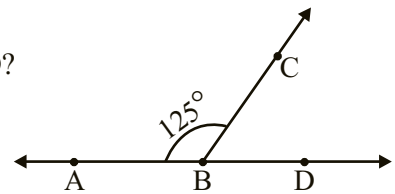
3. Use an equation to find the supplement of the following angles. Show your work.

a. $\angle X = 37°$ _____

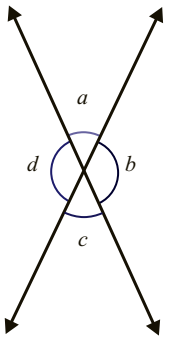
b. $\angle Y = 64°$ _____

c. $\angle Z = 108°$ _____

4. $\angle ABC$ and $\angle CBD$ are supplementary. What is the measure of $\angle CBD$?



5. Which pairs of angle are supplementary? Explain how you know.



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6. Two angles are supplementary. One angle is five times larger than the other.
What are the measures of the two angles?

---

7. Jeremy measures an angle that is 40° more than its supplement.
Find the supplement of the angle measured by Jeremy.

---

8. Two angles are supplementary. One angle is 50° more than the other.
Find the measures of the two angles.

---

9. Andre is is trying to draw a circle graph for a class presentation.
He needs to draw two supplementary angles, one four times as large as the other.
Use a protractor to draw the angles Andre should draw on his circle graph. Show your work.

---

10. Rhonda has designed a new piece of jewelry. The piece is a semi-circle made up of two supplementary angles. The smaller of the angles is a third of the size of the larger angle.
Draw the angles on the piece of jewelry. Show your work.

Name _____ Class _____ Date _____

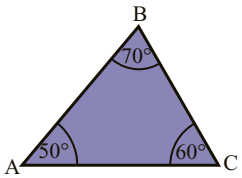
Lesson 7.2.4

The Triangle Sum

California Standard: Measurement and Geometry 2.2

Example

What is the sum of the three angles of the triangle?

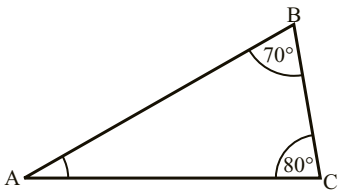


Solution

180°

The sum of the three angles in any triangle will always be 180°.

1. Find the measure of ∠A?



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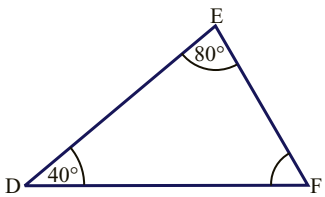
2. Use an equation to find the measure of ∠F.
Show your work.



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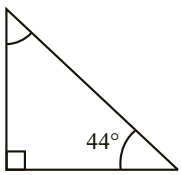
3. Find the measure of the missing angle in each of the triangles below.

a.



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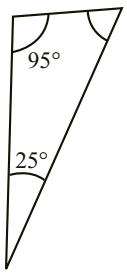
b.



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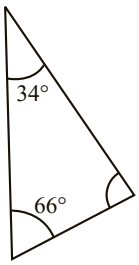
c.



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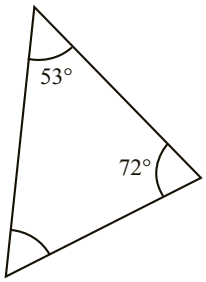
d.



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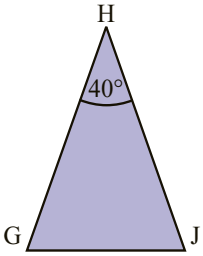
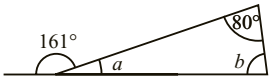
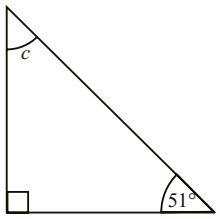
4. GHJ is an isosceles triangle.
Find the measures of ∠G and ∠J.

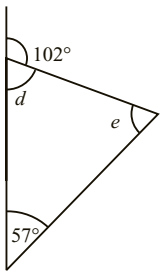


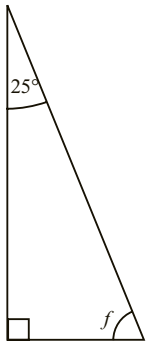
Remember — in an isosceles triangle, two of the angles are equal.

5. Find the measures of the missing angles $a - f$ in the following triangles.







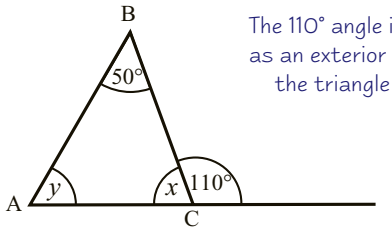


---

6. The sum of the measures of two angles in a triangle is 110°.

What is the measure of the third angle? ---

7. Find the measures of angles x and y in the triangle shown.



The 110° angle is known as an exterior angle of the triangle ABC.

---

8. David draws the triangle ABC. $\angle A$ is three times larger than $\angle B$, and $\angle C = 40°$.
Find the measures of angles at A and B.

---

9. Geoff uses some wire to hang a picture on his wall. When the picture is hung, the wire forms a triangle with two equal angles. The angle at the top of the triangle is 60° larger than each of the two equal angles. Find the measures of the angles in the triangle of wire.

---

10. Carla has just bought a three-legged stool. A line drawn on the floor around the legs of the stool creates a triangle with two equal angles, plus another angle of 75°.
What are the measures of the other two angles?

Name _____ Class _____ Date _____

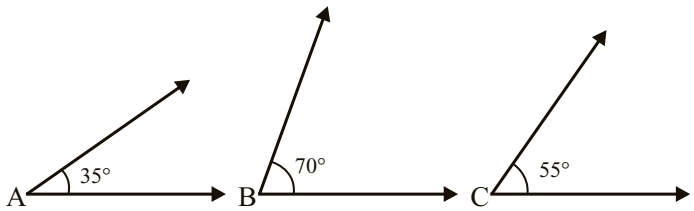
Lesson 7.2.5

Complementary Angles

California Standard: Measurement and Geometry 2.2

Example

Which of the following angles are complementary?



Complementary angles are two angles whose sum of the measures is 90°.

Solution

Since ∠A + ∠C = 90°, they are complementary angles.

1. State whether the following pairs of angles are complementary or not complementary.

a. 40°, 50°

---

b. 30°, 100°

---

c. 10°, 80°

---

d. 20°, 80°

---

2. Find the complement of each of the following angles.

a. ∠Z = 45° ---

b. ∠Y = 33° ---

c. ∠X = 89° ---

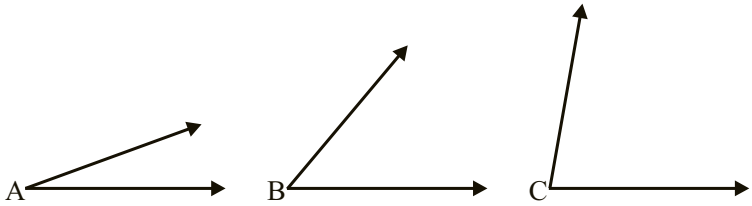
d. ∠W = 78° ---

3. Use your protractor to measure each angle, then state its complement.

Complement to ∠A = _____

Complement to ∠B = _____

Complement to ∠C = _____



4. Given that $\angle P = 33°$, write and solve an equation that will find its complement.

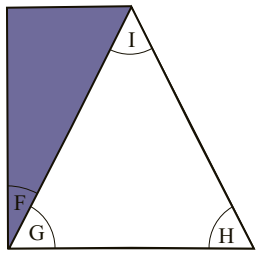
---

5. The sum of which of the following angles would **not** produce a right angle?

- ☐ 35° and 55°
- ☐ 61° and 29°
- ☐ 27° and 73°
- ☐ 48° and 42°

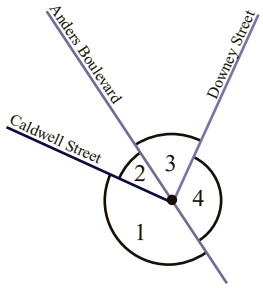
6. $\angle R = 55°$, and $\angle R$ and $\angle S$ are complementary. What is the measure of $\angle S$? ---

7. Juan is putting two pieces of tile into a mosaic.
$\angle G$ and $\angle H$ have the same measure and $\angle F$ and $\angle G$ are complementary.
If the measure of $\angle F = 24°$, find the measures of angles G,H, and I.

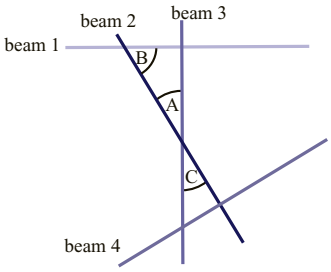


8. The intersection of several streets is shown on the map on the right.
Anders Boulevard runs in a straight line.
Caldwell Street and Downey Street meet at 90 degrees to each other.
If the measure of $\angle 1 = 148°$, find the measures of the following angles.

$\angle 2$ ---------- $\angle 3$ ---------- $\angle 4$ ----------



9. At the latest movie premier, 4 search lights were used to make a pattern of light beams in the sky. Beams 1 and 3 and beams 2 and 4 are at right angles to each other, and $\angle A = 31°$.
Find the measure of angles B and C.



Name _____ Class _____ Date _____

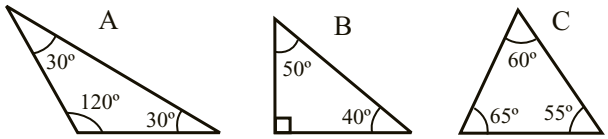
Lesson 7.3.1

Classifying Triangles by Angles

California Standard: Measurement and Geometry 2.3

Example

Classify each triangle by the measure of its angles.



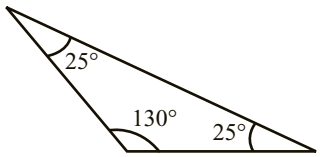
Solution

A — obtuse triangle B — right triangle C — acute triangle
Triangle 'A' has one obtuse angle, so it is an obtuse triangle.
Triangle 'B' has one right triangle, so it is a right triangle.
Triangle 'C' has all angles acute, so it is an acute triangle.

Recall that the sum of the measures of the angles of a triangle equals 180°.

1. Which of the following is the correct classification for the triangle shown?

☐ acute ☐ obtuse ☐ right ☐ linear



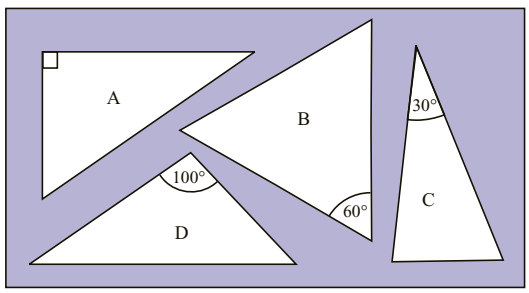
2. Classify each of the triangles below by the measure of its angles.

A _____

B _____

C _____

D _____



3. Classify each of the following triangles given the measures of its angles.

a. 36°, 70°, 74°

_____

b. 111°, 39°, 30°

_____

c. 56°, 90°, 34°

_____

d. 95°, 38°, 47°

_____

4. Classify each of the following triangles given the measure of two of its angles.

a. 50°, 60° **b.** 37°, 53° **c.** 24°, 46°

---

5. Two angles of a triangle measure 35° and 55°. What type of triangle is it?

---

6. Can a triangle have two right angles? Explain your answer.

---

7. Using your protractor, construct triangles whose angles measure the following. Classify each triangle.

a. 45°, 45°, 90° **b.** 30°, 50°, 100° **c.** 50°, 60°, 70°

---

8. Use your protractor to construct the following triangles. Give the measure of the third angle, and classify the triangles.

a. 20°, 50° **b.** 40°, 50° **c.** 50°, 70°

---

9. Janice said, "I have drawn a triangle that has one acute angle, one obtuse angle and one right angle." Explain why Janice must be wrong.

Name _____ Class _____ Date _____

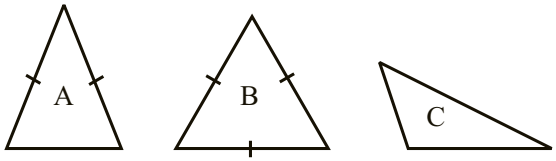
Lesson 7.3.2

Classifying Triangles by Side Length

California Standard: Measurement and Geometry 2.3

Example

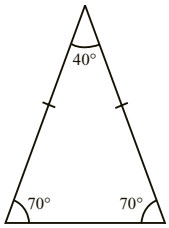
Classify each of the following triangles by the length of their sides.



Solution

A — **isosceles**, because exactly two sides have the same length.
B — **equilateral**, because all three sides have the same length.
C — **scalene**, because none of the sides have the same length.

1. Which of the following is the correct classification for the triangle shown?



☐ scalene ☐ isosceles ☐ equilateral ☐ linear

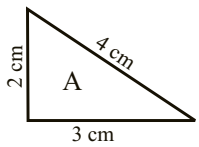
---

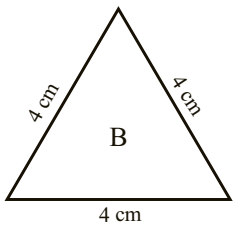
2. Classify each of the following triangles by their side lengths.

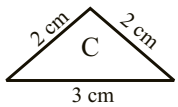
A _____

B _____

C _____







3. Classify triangles with the following side lengths as isosceles, equilateral, or scalene.

a. 7 in, 4 in, 7 in _____

b. 3 cm, 4 cm, 5 cm _____

c. 6 ft, 6 ft, 6 ft _____

4. Rachel is weaving a basket with a triangular pattern.
Each of the triangles in the pattern has two 2.5 cm sides and one side 3 cm in length.

a. Use a compass and ruler to draw a triangle the same size as those in the pattern.

b. Classify this triangle by its side lengths. _____

5. Tim is cutting out equilateral triangles from a strip of card to make a crown. Each of the triangles has sides of length 2.3 cm. Use your compass and ruler to draw one of these triangles.

6. Melanie is dressing up as a cat for halloween. To make the ears for her costume, she cuts out two equilateral triangles, each having a side length of 2 inches, and wants to outline them using ribbon.

a. Use a compass and ruler to draw a triangle with the same dimensions as one of Melanie's cat ears.

b. How much ribbon would Melanie need to outline both ears? ----------

7. Using your compass and ruler, draw the following triangles:

a. Equilateral triangle with side lengths 3 cm.

b. Scalene triangle with side lengths 2 cm, 3 cm, 4 cm.

c. Isosceles triangle with side lengths 2 cm, 3 cm, 3 cm.

8. Use your compass and ruler to construct a scalene triangle with sides measuring 4.5 cm, 3.2 cm and 2.4 cm.

9. Is it possible to draw a triangle whose sides measure 2 in, 3 in, and 6 in? Explain your answer.

---

10. Joel said, "I need to draw a triangle with its two shortest sides measuring 6 cm and 9 cm."

What can you say about the length of the third side of this triangle? ----------

Name _____ Class _____ Date _____

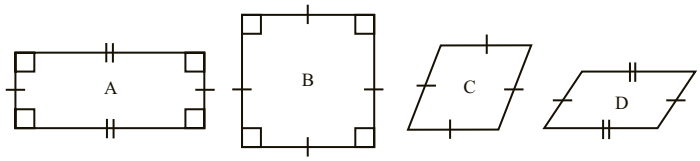
Lesson 7.3.3

Types of Quadrilaterals

California Standard: Measurement and Geometry 2.3

Example

Name each of the following figures.



Solution

A is a rectangle and parallelogram
B is a square, rectangle, and parallelogram
C is a rhombus and parallelogram
D is a parallelogram
All are parallelograms because they have two pairs of parallel sides.
A is a rectangle because it has 4 right angles.
B is a square because it has all sides the same length and 4 right angles, so it is also a rectangle.
C is a rhombus because it has all sides the same length, but no 90° angles.

1. Complete the following sentences by filling in the blank with the correct word.

a. A parallelogram has exactly _____ pairs of of sides that are parallel.

b. A parallelogram with four right angles is called a _____ .

2. Classify each of the following. If they have more than one classification, list all of them.

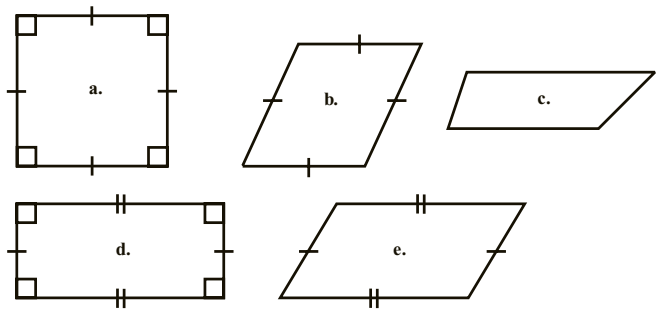
a. _____

b. _____

c. _____

d. _____

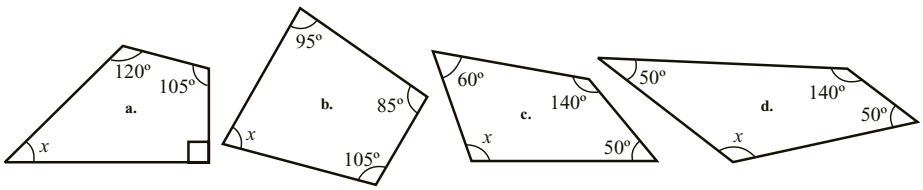
e. _____



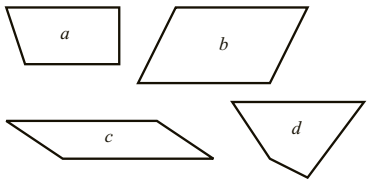
3. Determine the measure of angle x in each of the figures below.

a. _____ **b.** _____

c. _____ **d.** _____



4. Which of the following are parallelograms?



---

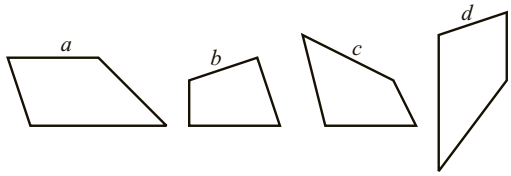
5. What type of quadrilateral is being described below?
"I have two pairs of parallel sides.
All my angles are equal, and all of my sides are equal in length."

---

6. Is the following figure a trapezoid? Explain your answer.

---

7. Which of the following quadrilaterals are trapezoids?



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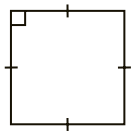
8. This figure can be classified as a:

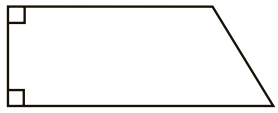


a. _____, **b.** _____,

c. _____, **d.** _____.

9. Draw a trapezoid where the bases measure 3 cm and 2 cm.

10. Which of the following is the correct classification for this shape?



☐ parallelogram ☐ square

☐ trapezoid ☐ rhombus

11. What is the sum of the interior angles of a rhombus?
Explain your answer.

---

12. A 4-sided figure has base angles of 60°, with the sides adjoining the base both 6 cm long.
What type of polygon is the figure?

Name _____ Class _____ Date _____

Lesson 7.3.4

Drawing Quadrilaterals

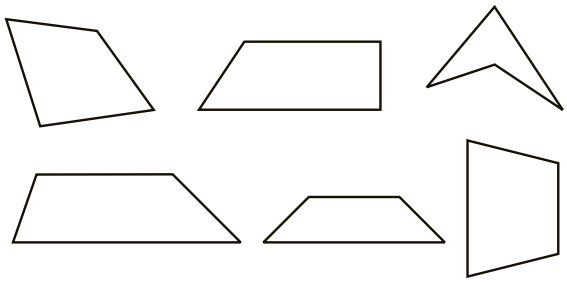
California Standard: Measurment and Geometry 2.3

Example

Draw at least three different quadrilaterals.

Solution

Various answers are possible — six examples are shown to the right.
Any three 4-sided figures will be correct.



Any quadrilateral with exactly one pair of parallel sides is called a trapezoid.

1. Describe how to draw a rectangle when the length of all the sides is known.

2. Using your ruler and protractor, draw a parallelogram with sides 5 cm and 2 cm.

3. Where possible, draw a parallelogram with the following angles.

a. 50°, 130°

b. 75°, 105°

c. 30°, 150°

d. 80°, 120°

4. Corbin is drawing an isosceles trapezoid for his homework.
He knows that the length of one of the bases of the trapezoid should be 4.5 cm.
What other information does Corbin need to draw the rest of the shape?

---

5. Raul is drawing a parallelogram by using supplementary angles. He knows that one angle is 45°.
What other information does Raul need to draw the rest of the parallelogram?

---

6. Yolanda is making a mosaic design by cutting out shapes from wrapping paper.
She needs to cut out the following shapes. Draw each shape and label the lengths of its sides.

a. An isosceles trapezoid with 2-inch and 1-inch bases.

b. A parallelogram with 1-inch and 0.5-inch sides and a 95° angle.

c. A rectangle whose length is 3 times its width.

d. A trapezoid with base lengths 5 cm and 2 cm, on top of a 5 cm × 3 cm rectangle.

Name _____ Class _____ Date _____

Lesson 7.4.1

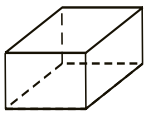
Three-Dimensional Figures

California Standard: Measurement and Geometry 1.3

Example

How many faces, edges and vertices does this rectangular prism have?

The top and bottom must be parallel to each other to have a prism.



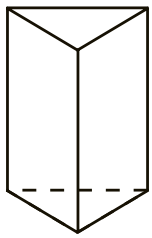
Solution

6 faces — 4 sides, the top, and the bottom.
12 edges — 4 around the top, 4 around the bottom, and 4 connecting the top and bottom.
8 vertices — 4 on the top, 4 on the bottom (where edges meet).

1. Tick the box next to the correct name of the figure shown.

rectangular prism ❒ triangular prism ❒

cylinder ❒ cube ❒



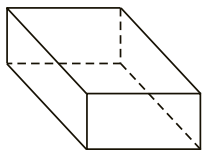
2. Name each of the following three dimensional figures.

a. _____

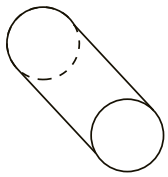
b. _____

c. _____

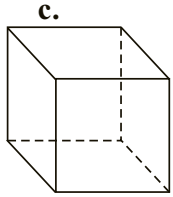
a.



b.



c.



3. a. Given the rectangular prism to the right, use letters to denote the:

faces _____ _____ _____

_____ _____ _____

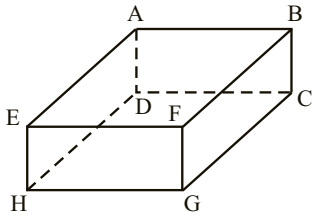
edges _____ _____ _____ _____

_____ _____ _____ _____

_____ _____ _____ _____

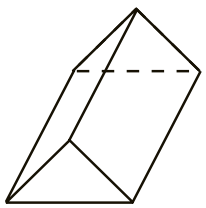
vertices _____ _____ _____ _____

_____ _____ _____ _____

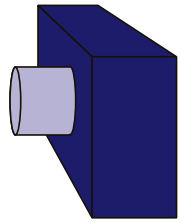


b. How many faces, edges, and vertices does the prism to the right have?

faces _____ edges _____ vertices _____



4. Charlene has made a model camera by folding cardstock into two different shapes. Name the two types of shapes Charlene made to form the camera.



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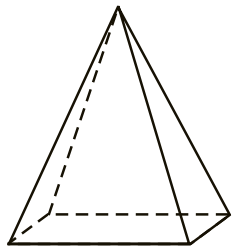
5. Explain why a pyramid is **not** a prism.



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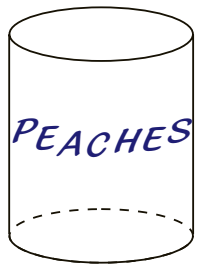
6. Explain why this can of fruit is not a prism.
What is the name of this shape?



---

7. Jason's mailbox has three more edges than vertices. What type of prism is Jason's mailbox?

---

8. Dimitri is making models of prisms using straws (for the edges) and marshmallows (for the vertices). How many straws and marshmallows would he need to make 3 triangular prisms?

Name _____ Class _____ Date _____

Lesson 7.4.2

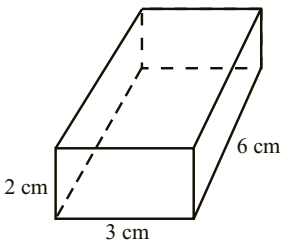
Volumes of Rectangular Prisms

California Standard: Measurement and Geometry 1.3

1. Draw a cube with a volume of 8 cm^3.

Example

Find the volume of this prism.



Solution

Volume (V) = Bh where B is the area of one of the prism's bases, and h is the height of the prism.

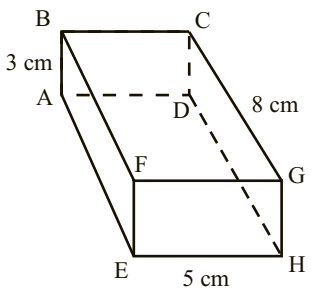
$V = (3 \times 2) \times 6$

$V = 6 \times 6$ The area of a rectangle = length × width

$V = 36$

So the volume of the prism is **36 cm^3**.

2. The diagram shows a rectangular prism.



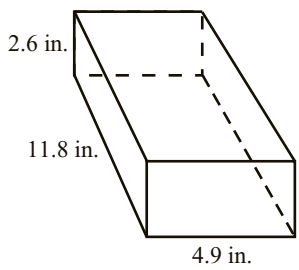
a. Identify two bases and the height for the rectangular prism shown.

---

b. Which of the following is the volume of this prism? Tick the correct box.

☐ 15 cm^3 ☐ 24 cm^3 ☐ 40 cm^3 ☐ 120 cm^3

3. Calculate the volume of this rectangular prism.
Round your answer to the nearest tenth.



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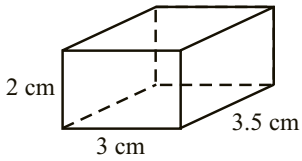
4. Find the volume of each rectangular prism below.

a.



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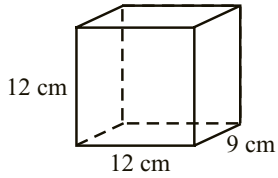
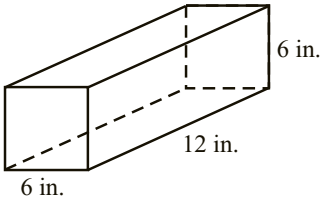
b.



---

5. Andrea takes her lunch to school in the box shown below. What is the volume of Andrea's lunchbox?



---

6. A cube has a length of 8 in. What is the width and the height of the cube?

---

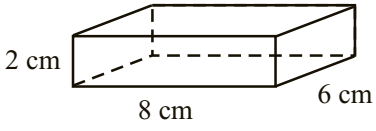
7. Which of the following is the volume of a cube with an edge of 20 inches? Tick the correct box.

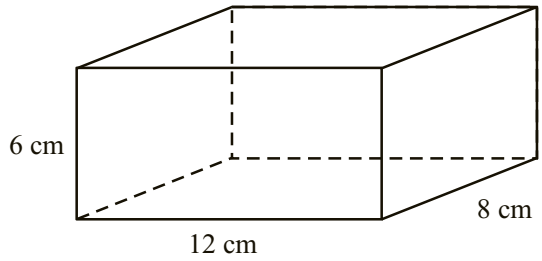
☐ 400 in^3 ☐ 4000 in^3 ☐ 800 in^3 ☐ 8000 in^3

8. What happens to the volume of a cube when you double the length of its sides?

---

9. Reggie's Chinese take-out restaurant packs food into the two sizes of box shown.





What is the difference in the volume between the two boxes?

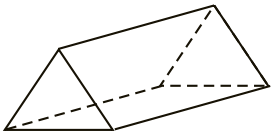
Name _____ Class _____ Date _____

Lesson 7.4.3

Volumes of Triangular Prisms and Cylinders

California Standard: Measurement and Geometry 1.3

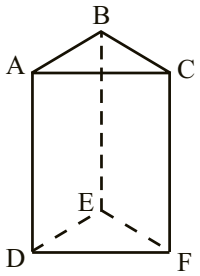
1. Shade the bases of this triangular prism.



2. Use letters to denote the bases and the height of the triangular prism shown.

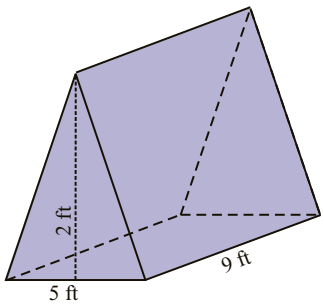
Bases: _____ and _____

Height: _____



Example

Find the volume of the triangular prism below.



Solution

Volume = Bh, where B is the area of one of the prism's bases and h is the height of the prism.

The bases of this prism are triangles so:

$V = (\frac{1}{2} \times 2 \times 5) \times 9$

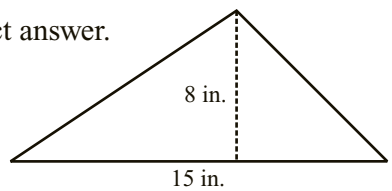
The area of a triangle $= \frac{1}{2} \times$ base $\times$ height.

$V = 5 \times 9$

$V = 45$

So the volume of the prism is **45 ft³**.

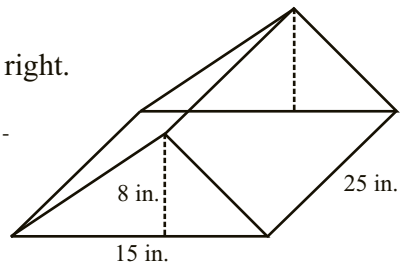
3. a. What is the area of this triangle? Tick the box next to the correct answer.



☐ 120 in² ☐ 60 in²

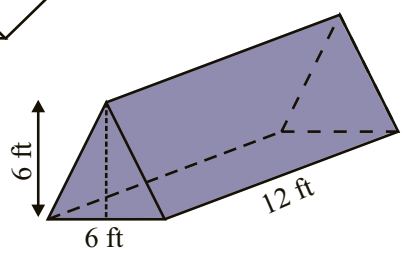
☐ 30 in² ☐ 15 in²

b. Find the volume of the triangular prism to the right.



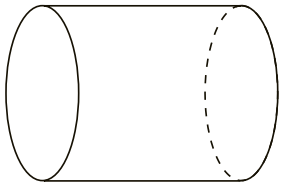
_____

4. Jackie's father is building her a play tent with the dimensions shown in the diagram. Find the volume of the play tent.



_____

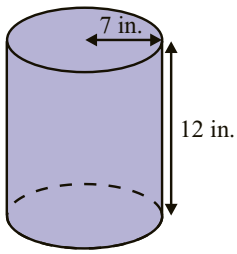
5. Shade the bases of this cylinder.



6. Find the approximate area of a circle with a 14 inch diameter.
Use $\pi \approx 3.14$.

---

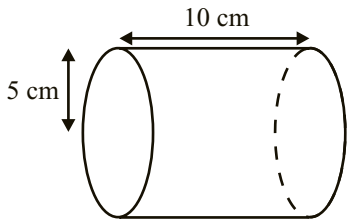
7. Tick the box next to the approximate volume of this cylinder. Use $\pi \approx \frac{22}{7}$.



☐ 3696 in^3 ☐ 1848 in^3

☐ 924 in^3 ☐ 462 in^3

8. Using $\pi \approx 3.14$, find the volume of this cylinder.



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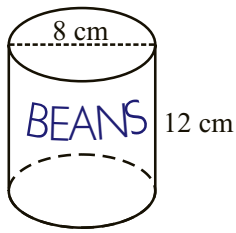
9. Enrique is cooking beans on toast for his lunch.
What is the approximate volume of the can of beans shown? Use $\pi \approx 3.14$.



---

10. The distance between the bases of a triangular prism is 15 in. It has a volume of 360 in^3.
What is the area of each base of the prism?

---

11. Mrs. Spence is considering buying a circular swimming pool.
She can't decide between two pools, Pool A and Pool B.
She wants to buy the pool that will hold the most water.
Which pool should she buy?

Pool A:
diameter = 36 ft, depth = 4 ft.
Pool B:
diameter = 42 ft, depth = 3 ft.

---

12. Mr. Lopez is concerned about the cost of the water if he builds a pool.
He is thinking of building a 4-foot-deep circular pool, with a diameter of 25 feet.
He works out it would cost 0.25 cents per cubic foot to fill the pool. Tick the correct box to show the cost of filling this pool to the nearest dollar. (Use $\pi \approx 3.14$.)

☐ \$290 ☐ \$390 ☐ \$490 ☐ \$590

Name _____ Class _____ Date _____

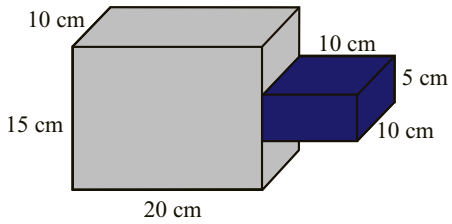
Lesson 7.4.4

Volumes of Compound Solids

California Standard: Measurement and Geometry 1.3

Example

Find the volume of this figure.

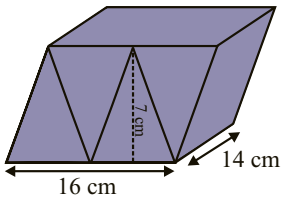


Solution

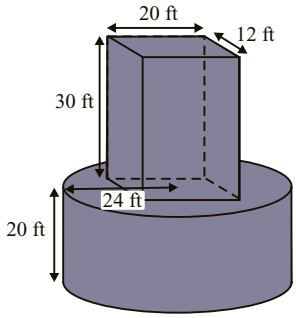
The figure is made up of two rectangular prisms.
Find the volume of each prism and then add them together.
The volume of the first prism is:
$V = Bh = (l \times w) \times h = 10 \times 20 \times 15 = 3000 \text{ cm}^3$.
The volume of the second prism is:
$V = Bh = (l \times w) \times h = 10 \times 10 \times 5 = 500 \text{ cm}^3$.
So the volume of the prism is 3000 + 500 = **3500 cu cm** or **3500 cm^3**.

1. Find the volume of each of the figures below. Use $\pi \approx 3.14$.

a.



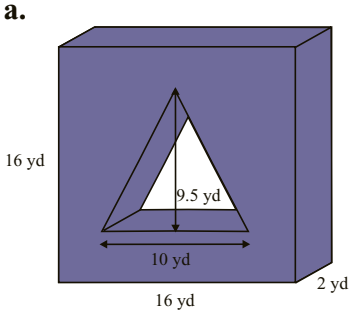
b.



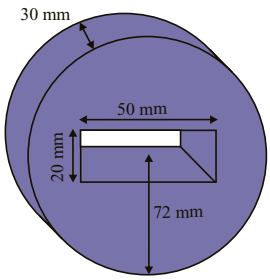
2. Describe how the volume of a cylinder would change if a triangular prism was cut out of the middle of it.

3. Find the volume of each of the figures below. Use $\pi \approx 3.14$.

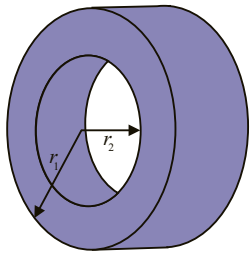
a.



b.



4. Mailka wants to find the volume of the hollow tube shown.
She says, "The tube can be broken down into two cylinders.
I can use the difference between the radii of the cylinders ($r_1 - r_2$) as the value for radius in the equation $V = \pi r^2 h$ to find the volume of the tube."
Will this calculation give her the correct volume of the tube?
Explain your answer.



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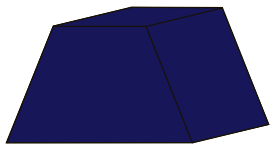
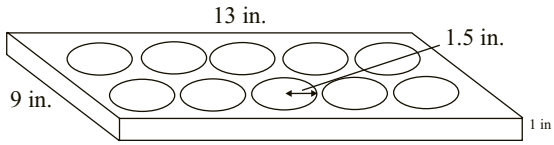
5. Josephine worked out the volume of a triangular prism. She then cut off the top of the prism to make the figure shown, and threw the top piece in the trash.
How could she work out the volume of the missing piece only using the prism left behind?



---

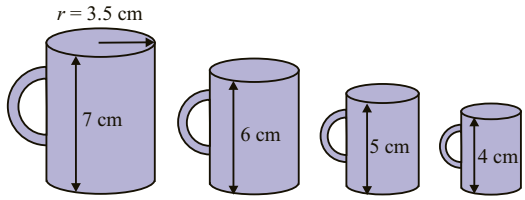
6. A baker rolled out some cookie dough in the shape of a rectangular prism.
Using a cylindrical cutter, he cut 10 cookies out of the dough as shown in the figure below.



What volume of dough was left after all 10 cookies were cut out and removed?
Use $\pi \approx 3.14$.

---

7. Marcie has 4 cylindrical measuring cups as shown below.
The largest cup had a radius, $r = 3.5$ cm. The radius of each smaller cup is 0.5 cm less than the next largest cup.
What is the total volume of Marcie's measuring cups?
Use $\pi \approx 3.14$.



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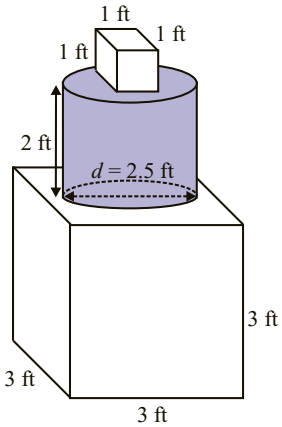
8. Some building blocks were used to make the sculpture to the right.
What is the total volume of the sculpture, to the nearest cubic foot?
Use $\pi \approx 3.14$.



Name _____ Class _____ Date _____

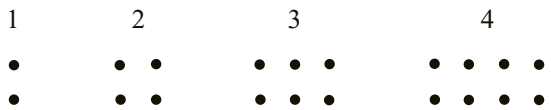
Lesson 7.5.1

Generalizing Results

California Standards: Mathematical Reasoning 1.2, 3.1, 3.2, 3.3

Example

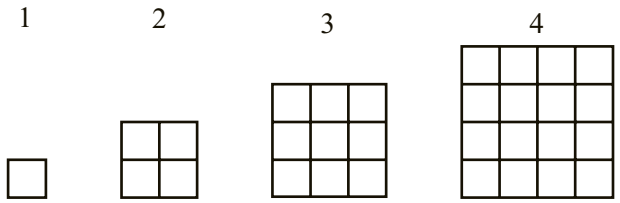
How many dots will make up the 8th figure in the pattern shown?



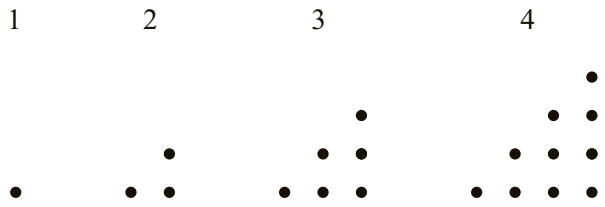
Solution

For each figure, the number of dots is double the number of the figure. This can be written as $2n$, where n is the number of the figure. So the 8th figure will have $8 \times 2 = 16$ dots.

1. In the pattern below, how many 1-by-1 squares will make up the 10th figure? _____



2. Josie is studying a pattern of dots, and has drawn the four figures below.



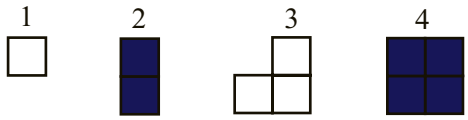
a. How many dots would be in figure 5? _____

b. How many dots would be in figure 8? _____

c. Which of the following is a rule that will give the number of dots for any figure? 'n' is the number of the figure.

❒ $2n$ ❒ $\frac{1}{2}(n^2 + n)$ ❒ $2n + 2$ ❒ $\frac{1}{2}n^2$

3. Freddie made this pattern using cubes.



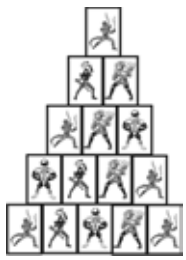
a. What color would the cubes in the 88th figure be?

---

b. Freddie has just enough cubes to complete the pattern up to figure 6.
How many white cubes does Freddie have altogether?

---

4. Dana is storing her trading cards as shown in the diagram.
How many cards would Dana have in the 18th row?
Explain your answer.



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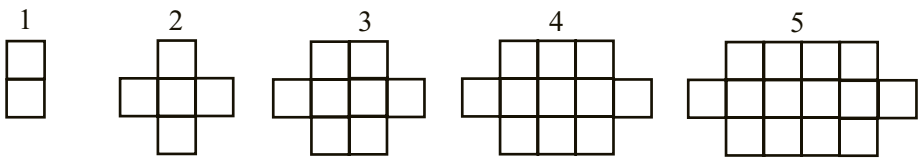
5. Given the following pattern, how many blocks would be needed to make:



a. the 7th figure? _____ **b.** the 10th figure? _____

6. The number of revolutions of a wheel and the distance the wheel travels is shown in the table below.

Number of revolutions	Distance traveled (inches)
1	82
2	164
3	
4	

a. Complete the table.
b. Write an expression that can be used to find the distance, d, covered by n revolutions.

---

c. How many times would the wheel need to revolve to cover 24,600 inches?

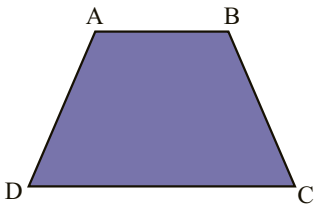
Name _____ Class _____ Date _____

Lesson 7.5.2

Proving Generalizations

California Standards: Mathematical Reasoning 1.2, 3.2, 3.3

1. Decide whether each of the following statements on isosceles trapezoids is true or false.



a. All isosceles trapezoids have at least two sides with the same length.

---

b. The top base of all isosceles trapezoids is less than half the length of the bottom base.

---

2. Julian draws a cylinder with a volume of x in^3, which has a base area of y in^2.
Decide whether each of the following statements about the cylinder is true or false.

a. For any cylinder, x is always larger than y. _____

b. Cutting a cylinder along a diameter of its bases is the same as halving the cylinder. _____

Example

Is it possible to have a parallelogram that contains a right angle? Explain your answer.

Solution

Yes it is. A parallelogram has to have two pairs of parallel sides and the angles on any side must equal 180°. So all angles in the parallelogram must be 90° — a rectangle.

3. Michelle said, "The faces of a triangular prism cannot be square."
Is she right? Explain your answer.

---

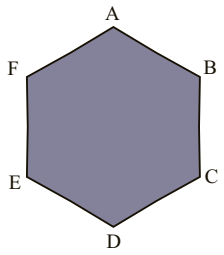
4. Cory said, "In order to prove a statement to be incorrect, only one counterexample is needed."
Is he right? Explain your answer.

---

5. Alyssa said, "If two quadrilaterals have the same angles as each other, they must be similar."
Is this statement true? Explain your answer.

---

6. The floor of the new school hall is shaped like a hexagon.
All sides of the hexagon are equal in length.
Decide whether each of the following statements is true or false.
In each case, explain your answer.



a. The sum of the interior angles of the hexagonal floor must be 360°.

---

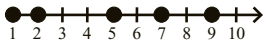
b. The maximum number of diagonals that can be drawn on the floor is 9.

---

c. The sum of the angles in a hexagon is greater than the sum of the angles in a quadrilateral.

Section 1.1

Lesson 1.1.1 — Comparing Integers

1. a. 

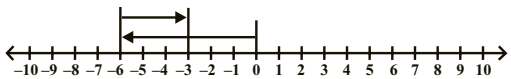
   b. 9
   c. 1
   d. 9
   e. 1

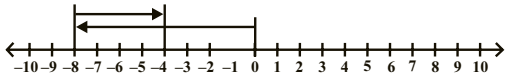
2. a. 

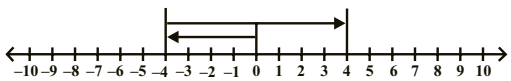
   b. 6
   c. 0
   d. 6
   e. 0

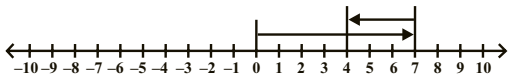
3. a. 2, 7, 5, 1
   b. 2, 7, 0, 5, 1
   c. 2, 7, 0, 5, 1

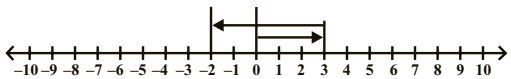
Lesson 1.1.2 — Adding and Subtracting Integers

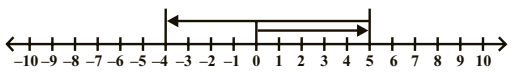
1. a. $-6 + 3 = \mathbf{-3}$


   b. $-8 + 4 = \mathbf{-4}$


   c. $-4 + 8 = \mathbf{4}$


   d. $7 + (-3) = \mathbf{4}$


   e. $3 + (-5) = \mathbf{-2}$


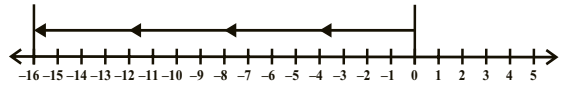
   f. $5 + (-9) = \mathbf{-4}$


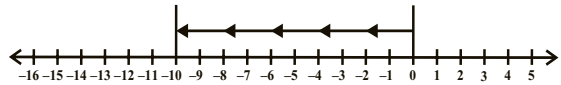
2. a. $-4 + 7 = \mathbf{3}$ b. $5 + (-4) = \mathbf{1}$
   c. $-3 + 8 = \mathbf{5}$ d. $7 + (-2) = \mathbf{5}$
   e. $-7 + 11 = \mathbf{4}$ f. $-5 + 6 = \mathbf{1}$
   g. $6 + (-4) = \mathbf{2}$ h. $9 + (-6) = \mathbf{3}$
   i. $8 + (-6) = \mathbf{2}$ j. $-8 + 11 = \mathbf{3}$
   k. $8 + (-2) = \mathbf{6}$ l. $-5 + 10 = \mathbf{5}$

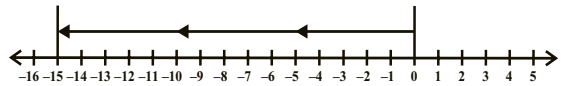
3. $7 + (-5) + 4 + 3 + (-2) + (-6) = 1$,
   so Pablo **gained 1 yard** overall in this game.

Section 1.2

Lesson 1.2.1 — Multiplying with Integers

1. a. $4 \times (-4) = \mathbf{-16}$


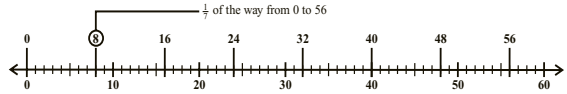
   b. $5 \times (-2) = \mathbf{-10}$


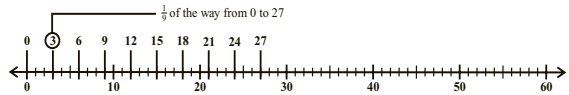
   c. $3 \times (-5) = \mathbf{-15}$


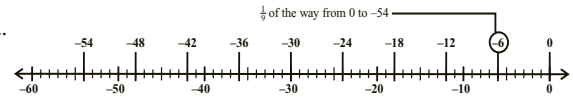
2. b. $\mathbf{4 \times (-3) = -12}$
   c. $\mathbf{3 \times (-5) = -15}$

3. a. $4 \times (-3) = \mathbf{-12}$ b. $-4 \times (-6) = \mathbf{24}$
   c. $-9 \times 8 = \mathbf{-72}$ d. $6 \times (-9) = \mathbf{-54}$
   e. $8 \times 7 = \mathbf{56}$ f. $-5 \times 9 = \mathbf{-45}$
   g. $7 \times (-9) = \mathbf{-63}$ h. $-4 \times (-9) = \mathbf{36}$
   i. $9 \times (-3) = \mathbf{-27}$ j. $-6 \times (-8) = \mathbf{48}$
   k. $-8 \times 4 = \mathbf{-32}$ l. $3 \times 8 = \mathbf{24}$
   m. $-6 \times 7 = \mathbf{-42}$ n. $7 \times (-4) = \mathbf{-28}$
   o. $-3 \times (-7) = \mathbf{21}$ p. $-6 \times 4 = \mathbf{-24}$

Lesson 1.2.2 — Dividing with Integers

1. a. $56 \div 7 = 8$


   b. $27 \div 9 = 3$


2. a.


   b. $-54 \div 9 = -6$, as shown in part a. So $-54 \div (-9) = \mathbf{6}$.

3. b. $36 \div 3 = 12$
   c. $36 \div 4 = 9$
   d. $36 \div 6 = 6$

Lesson 1.2.3 — Integers in Real Life

1. a. –
   b. +
   c. ÷
   d. ×

2. a. **4**, the number Mrs. Smith will divide by.
   b. **24**, the number Mrs. Smith will divide.
   c. The answer, 24 ÷ 4 = **6**
   d. **6 pencils**

3. –120 ÷ 6 = –20, so the son is **20 feet** below the surface.

Section 1.3

Lesson 1.3.1 — Decimals

1. a. 8
   b. 3
   c. 3
   d. 5

2. 3406.5802

3. a. 0
   b. 9
   c. 2
   d. 3
   e. 4

Lesson 1.3.2 — Ordering Decimals

1. a. 34.59 > 34.56 because it is to the right on the number line, so **Fran** earned the most.
   b. 34.59 – 34.56 = 0.03 or **3 cents** more.

2. –7.5 < –7.3 < –7.1
   a. **Tuesday** morning was the coldest.
   b. **Wednesday** morning was the warmest.

3. a. 4.56 < 4.57
   b. 6.01 > 6.001
   c. 20 + 4 + .3 + .05 = 24.35 > 24.33
   d. five and forty-two hundredths < five and forty-three hundredths.

Section 1.4

Lesson 1.4.1 — Rounding Numbers

1. a. To the nearest whole number:
   7.842 = 8, 7.389 = 7, 8.340 = 8, 8.5 = 9
   So **8.5** is greater than 8 when rounded to the nearest whole number.
   b. To the nearest hundredth:
   7.842 = 7.84, 7.389 = 7.39, 8.340 = 8.34, 8.5 = 8.50
   So **7.389** is less than 7.8 when rounded to the nearest hundredth.

2. a. 5290.00 = 5287.89 to the nearest **ten**.
   b. 5000.00 = 5287.89 to the nearest **thousand**.
   c. 5288.00 = 5287.89 to the nearest **whole number**.

3. a. The digit to the right of the tens place is 8.
   8 > 5, so round up to **100**.
   b. The digit to the right of the units place is 3.
   3 < 5, so round down to **98**.
   c. The digit to the right of the tenths place is 7.
   7 > 5, so round up to **98.4**.

Lesson 1.4.2 — Using Rounded Numbers

1. b. **1 h 51 min**
   It is not necessary to use the exact figure, as timetables are not expected to be accurate to the nearest second.
   If the time was rounded to the nearest hour, the timetable would be misleading, as it would be several minutes away from the real time.

2. a. The reporter will want to use a number that gives an idea of the size of the probe, but is easy to read.
   The reporter will probably round up to **1000 kg**, as this is a more eye-catching number.
   b. The engineer will need to use the **exact figure** to do precise calculations.
   c. Jenny should make the information on her poster accurate, but people reading it won't need to know the mass to the nearest 0.001 kg. So rounding to the **nearest kg** would be appropriate in this case.

3. a. 1200 ÷ 30 = 40
   40 is close to 38.36, so Tammy's answer is **reasonable**.
   b. 9000 + 400 = 9400
   9400 is not very close to 12871.4, so Tammy's answer is **unreasonable**.
   c. 18 – 0.7 = 17.3
   17.3 is not very close to 11.295, so Tammy's answer is **unreasonable**.

Lesson 1.4.3 — Estimation

1. a. Answers may vary, but estimates should be **about 85 to 90 students**.
   b. Answers may vary, but estimates should be **about 135 to 140 students**.

2. a. Answers may vary, but estimates should be **about 75 students**.
   b. Answers may vary, but estimates should be **about 125 students**.

3. a. Answers may vary, but estimates should be **about 7.50¢**.
   b. Answers may vary, but estimates should be **about 16.25¢**.
   c. Answers may vary, but should be the difference between the estimates given in parts a. and b., that is about 16.25¢ – 7.50¢ = **8.75¢**.

Lesson 1.4.4 — Using Estimation

1. Estimation is **not appropriate**.
   He needs the exact amount to put on the check.

2. The measurements need to be **precise**. If they are not, the glass could be too large or too small.

3. Chico only needs to **estimate** the amount of paint he will need. Paint is sold in cans, so he will not be able to buy an exact amount of paint.

Section 2.1

Lesson 2.1.1 — Variables

1. a. **His brother's age**.
   b. Answers may vary — example: b.
2. a. **Wanda's date of birth**.
   b. Answers may vary — example: w.
3. a. **The number of hours Felipe worked yesterday**.
   b. Answers may vary — example: h.

Lesson 2.1.2 — Expressions

1. $f + 25$
2. $R \times 2 = 2R$
3. $15 \times P = 15P$

Lesson 2.1.3 — Multi-Variable Expressions

1. a. The product of z and x, or z multiplied by x.
   b. u divided by v, or the quotient of u and v.
   c. The product of 5 and r, less the product of 3 and s .
   d. The product of u and v, less w.
2. a. 2
   b. 2
   c. 1
   d. 3
   e. 3
   f. 4
3. a. 2 variables, 1 term
   b. 2 variables, 2 terms
   c. 3 variables, 3 terms
   d. 1 variable, 2 terms

Lesson 2.1.4 — Order of Operations

1. a. Exponents
   b. No, before Wendell multiplies by the 2 outside the parentheses, he must evaluate everything inside the parentheses.
2. a. $5 + 6 - 7 = 11 - 7 =$ **4**
   b. $3 + 4 \times 5 - 6 = 3 + 20 - 6 = 23 - 6 =$ **17**
   c. $5 + 24 \div 6 - 8 = 5 + 4 - 8 = 9 - 8 =$ **1**
   d. $30 \div 5 \times 6 = 6 \times 6 =$ **36**
3. a. $9 \times 7 + 9 \times 9 + 9 \times 10 = 9(7 + 9 + 10)$
   b. $63 + 81 + 90 = 234$
   Jose earned $234 dollars.

Section 2.2

Lesson 2.2.1 — Equations

1. a. Let n = the unknown number
   $\boldsymbol{n + 23 = 57}$
   b. Let n = the unknown number
   $\boldsymbol{n + 36 = 94}$
   c. Let n = the unknown number
   $\boldsymbol{n + 47 = 66}$
   d. Let n = the unknown number
   $\boldsymbol{n + 43 = 87}$
2. a. $\boldsymbol{y = 18}$
   b. $\boldsymbol{y = 9}$
3. a. $2 \times 5 \times t = 50$
   Simplify to get $10t = 50$, which means $\boldsymbol{t = 5}$
   b. $(t + 6) + 10 = 40$
   Simplify to get $t + 16 = 40$, which means $\boldsymbol{t = 24}$

Lesson 2.2.2 — Manipulating Equations

1. a. Substitute 14 for $z \Rightarrow z + 8 = 14 + 8 =$ **22**
   b. Substitute 9 for $p \Rightarrow 72 \div p = 72 \div 9 =$ **8**
   c. Substitute 21 for $l \Rightarrow 2 \times l = 2 \times 21 =$ **42**
   d. Substitute 7 for $m - n \Rightarrow m - n + 2 = 7 + 2 =$ **9**
   e. Substitute 13 for $k + t \Rightarrow k + 9 + t = 13 + 9 =$ **22**
   f. Substitute 20 for $20v \Rightarrow 20v \div 20 = 20 \div 20 =$ **1**
2. a. Substitute 12 for $y + z$ to give: $\boldsymbol{12 - 6 = 6}$
   b. Substitute $-z$ for $5 + y$ to give: $\boldsymbol{3 \times (-z)}$
3. Left side was multiplied by 2, but right side had 2 added to it.

Lesson 2.2.3 — Solving + and – Equations

1. a. $x + 22 = 44$ Subtract 22 from both sides $\Rightarrow \boldsymbol{x = 22}$
   b. $55 + y = 92$ Subtract 55 from both sides $\Rightarrow \boldsymbol{y = 37}$
   c. $z + 5.8 = 11.4$ Subtract 5.8 from both sides $\Rightarrow \boldsymbol{z = 5.6}$
   d. $6.9 + w = 13.7$ Subtract 6.9 from both sides $\Rightarrow \boldsymbol{w = 6.8}$
   e. $u + 4.3 = -6.2$ Subtract 4.3 from both sides $\Rightarrow \boldsymbol{u = -10.5}$
   f. $x + 87 = 45$ Subtract 87 from both sides $\Rightarrow \boldsymbol{x = -42}$
   g. $-4.7 + y = -8.4$
   Subtract –4.7 from (add 4.7 to) both sides $\Rightarrow \boldsymbol{y = -3.7}$
   h. $56 + z = -45$ Subtract 55 from both sides $\Rightarrow \boldsymbol{z = -101}$
2. a. Add 15 to both sides: $\boldsymbol{m = 9 + 15 = 24}$
   b. Add 47 to both sides: $\boldsymbol{n = 99 + 47 = 146}$
   c. Add 7.3 to both sides: $\boldsymbol{p = 9.4 + 7.3 = 16.7}$
   d. Add 6.9 to both sides: $\boldsymbol{q = 2.5 + 6.9 = 9.4}$
   e. Add 4.7 to both sides: $\boldsymbol{r = 2.3 + 4.7 = 7}$
   f. Add 13.8 to both sides: $\boldsymbol{s = -19.4 + 13.8 = -5.6}$
   g. Add 10.7 to both sides: $\boldsymbol{t = -13.5 + 10.7 = -2.8}$
   h. Add –6.4 to both sides: $\boldsymbol{u = -5.8 + (-6.4) = -12.2}$
3. a. $\boldsymbol{x - 35 = 18}$
   b. $\boldsymbol{x - 25 = 21}$
   c. $\boldsymbol{x - 36 = 47}$
   d. $\boldsymbol{x - 26 = 43}$

Lesson 2.2.4 — Solving × and ÷ Equations

1. Penelope is right. $4x$ is the same as $4 \times x$, so to find x you must divide by 4, as Penelope correctly did, not subtract 4 as Clive tried.

2. a. $3s = 15$ Divide both sides by 3 $\Rightarrow$ $\boldsymbol{s = 5}$
   b. $5t = 75$ Divide both sides by 5 $\Rightarrow$ $\boldsymbol{t = 15}$
   c. $9u = 108$ Divide both sides by 9 $\Rightarrow$ $\boldsymbol{u = 12}$
   d. $15v = 255$ Divide both sides by 15 $\Rightarrow$ $\boldsymbol{v = 17}$
   e. $2.4w = 16.8$ Divide both sides by 2.4 $\Rightarrow$ $\boldsymbol{w = 7}$
   f. $-6x = 48$ Divide both sides by -6 $\Rightarrow$ $\boldsymbol{x = -8}$
   g. $-4.6y = -27.14$ Divide both sides by -4.6 $\Rightarrow$ $\boldsymbol{y = 5.9}$
   h. $5.7z = -22.8$ Divide both sides by 5.7 $\Rightarrow$ $\boldsymbol{z = -4}$

3. a. $7x = 42$
   b. $9x = 72$
   c. $8x = 56$
   d. $6x = 48$

Lesson 2.2.5 — Graphing Equations

1. **Add 52 to both sides**: $y - 52 + 52 = 7 + 52 \Rightarrow y = 59$.

2. **Divide both sides by 15**: $15y \div 15 = 60 \div 15 \Rightarrow y = 4$.

3. **Multiply both sides by 5**: $y \div 5 \times 5 = 90 \times 5 \Rightarrow y = 450$.

Section 2.3

Lesson 2.3.1 — Expressions About Length

1. a. Perimeter of room A $= x + 10 + x + 10 = \boldsymbol{2x + 20}$
   Perimeter of room B $= x + 12 + x + 12 = \boldsymbol{2x + 24}$
   b. Total $= 2x + 20 + 2x + 24 = 4x + 44$.
   The banner must be $(4x + 44)$ feet long.

2. a. Perimeter $= x + 9 + 14 = \boldsymbol{(x + 23)}$ **cm**
   b. Perimeter $= \boldsymbol{(12 + y + z)}$ **cm**

3. a. Perimeter $= 13 + 12 + 8 + 9 + a = \boldsymbol{(42 + a)}$ **cm**
   b. Perimeter $= 10 + b + c + c + c = \boldsymbol{(10 + b + 3c)}$ **cm**

Lesson 2.3.2 — Expressions About Areas

1. a. Area $= 7 \times 9$
   b. Area $= 4 \times x = 4x$
   c. Area $= y \times 7 = 7y$
   d. Area $= p \times q = pq$

2. a. Area $= b \times h = 4 \times 9 = \boldsymbol{36}$ **in²**
   b. Area $= b \times h = 7 \times 19 = \boldsymbol{133}$ **in²**
   c. Area $= b \times h = 5 \times 22 = \boldsymbol{110}$ **in²**
   d. Area $= b \times h = 14 \times 14 = \boldsymbol{196}$ **in²**

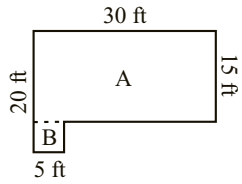
3. a. Area $= \frac{1}{2} \times b \times h = \boldsymbol{\frac{1}{2} \times 15 \times 8 = 60}$ **cm²**

   b. Area $= \frac{1}{2} \times b \times h = \boldsymbol{\frac{1}{2} \times x \times 3 = \frac{1}{2} \times 12 \times 3 = 18}$ **in²**

   c. Area $= \frac{1}{2} \times b \times h = \boldsymbol{\frac{1}{2} \times g \times h = \frac{1}{2} \times 7 \times 4 = 14}$ **yd²**

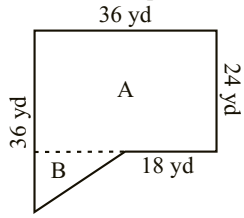
Lesson 2.3.3 — Finding Complex Areas

1. Answers may vary, but a possible answer is as follows:
   Divide the shape into two rectangles:



   Find the area of each rectangle.
   A: $30 \times 15 = 450$ ft²
   B: $5 \times 5 = 25$ ft²
   Total: A + B $= 450 + 25 =$ **475 ft²**

2. Answers may vary, but a possible answer is as follows:
   Divide the shape into a rectangle and a triangle:



   Find the area of each shape.
   A: $36 \times 24 = 864$ yd²
   B: $\frac{1}{2} \times (36 - 18) \times (36 - 24) = \frac{1}{2} \times 18 \times 12 = 108$ yd²
   Total: A + B $= 864 + 108 =$ **972 yd²**

Lesson 2.3.4 — The Distributive Property

1. $5(6 + 4) = 5 \times 6 + 5 \times 4 = 30 + 20 = \mathbf{50}$

2. a. $5(10 + 6) = 5 \times 10 + 5 \times 6 = 50 + 30 = \mathbf{80}$
   b. $7(20 + 11) = 7 \times 20 + 7 \times 11 = 140 + 77 = \mathbf{217}$
   c. $4(15 + 13) = 4 \times 15 + 4 \times 13 = 60 + 52 = \mathbf{112}$

3. a. $8(20 - 7) = 8 \times 20 - 8 \times 7 = 160 - 56 = \mathbf{104}$
   b. $9(30 - 8) = 9 \times 30 - 9 \times 8 = 270 - 72 = \mathbf{198}$
   c. $12(40 - 25) = 12 \times 40 - 12 \times 25 = 480 - 300 = \mathbf{180}$

Lesson 2.3.5 — Using the Distributive Property

1. a. $10(2 + 3) = 10 \times 5 = 50$
      $10 \times 2 + 10 \times 3 = 20 + 30 = 50$
   b. $5(6 + 9) = 5 \times 15 = 75$
      $5 \times 6 + 5 \times 9 = 30 + 45 = 75$
   c. $12(5 + 7) = 12 \times 12 = 144$
      $12 \times 5 + 12 \times 7 = 60 + 84 = 144$

2. a. $5(10 - 2) = 5 \times 10 - 5 \times 2 = 50 - 10 = \mathbf{40}$
   b. $-3(4 + 8) = -3 \times 4 + (-3) \times 8 = -12 + (-24) = \mathbf{-36}$
   c. $9(4 - (-6)) = 9 \times 4 - 9 \times (-6) = 36 - (-54) = \mathbf{90}$
   d. $-8(-3 + 2) = -8 \times (-3) + (-8) \times 2 = 24 + (-16) = \mathbf{8}$

3. **Pauline is wrong**. $17 \times 5 = 10 \times 5 + 7 \times 5 = 50 + 35 = 85$

Lesson 2.3.6 — Squares and Cubes

1. $6^3 = \mathbf{6 \times 6 \times 6 = 216}$

2. a. $7 \times 7 = 7^2 = \mathbf{49\ cm^2}$
   b. $12 \times 12 = 12^2 = \mathbf{144\ in^2}$
   c. $6 \times 6 = 6^2 = \mathbf{36\ yd^2}$
   d. $4 \times 4 = 4^2 = \mathbf{16\ ft^2}$

3. The area of each face is: $4 \times 4 = 4^2 = 16\ cm^2$.
   There are 6 faces in a cube, so the surface area is $6 \times 16 = \mathbf{96\ cm^2}$.

Lesson 2.3.7 — Expressions and Angles

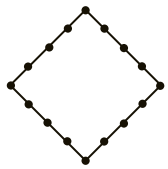
1. $(55 + y)°$

2. $(90 - x)°$

3. a. $p + 45 + 70 = \mathbf{p + 115 = 180}$
   b. $p = 180 - 115 = \mathbf{65}$

Section 2.4

Lesson 2.4.1 — Analyzing Problems

1. a.
   
   The triangle rotates 120° clockwise.
   b.
   
   The figure rotates 90° clockwise.
   c.
   
   The figure increases in size by one dot on each side.

2. a. 127, 255, 511. Mulitply the previous number by two and add one.
   b. 34, 55, 89. Sum the previous two numbers.
   c. 64, –128, 256. Multiplied the previous number by –2.
   d. 36, 49, 64. Square the position of the number in the sequence.
   e. $\frac{6}{7}, \frac{7}{8}, \frac{8}{9}$. Add one to the numerator and the denominator.

Lesson 2.4.2 — Important Information

1. a. The size of the angle.
   b. Where and how far Joe drives.
   c. The cost of frozen peas.

2. a. The cost of a small shirt.
   b. How much the tax is.
   c. What time Jim leaves his house.

3. a. Multiples of 9 between 50 and 100: 54, 63, 72, 81, 90
   Second digit less than 2: 81, 90
   Two possible numbers remain — **too little information**.
   b. The number is between 16 and 36.
   Multiples of 13 between 16 and 36: 26
   The number is determined with one clue to spare — **too much information**.
   c. The number is between 5 and 10.
   Multiples of 3 between 5 and 10: 6, 9
   With one digit: 6, 9
   Two possible numbers remain — **too little information**.

Lesson 2.4.3 — Breaking Up a Problem

1. Split the house into three rectangles and find the area of each:
   $40 \times (30 - 12) = 40 \times 18 = 720\ ft^2$
   $12 \times 10 = 120\ ft^2$
   $12 \times 12 = 144\ ft^2$
   Sum the area of each rectangle to find the total area:
   $720 + 120 + 144 = 984\ ft^2$
   $984\ ft^2$ of carpet will be needed.

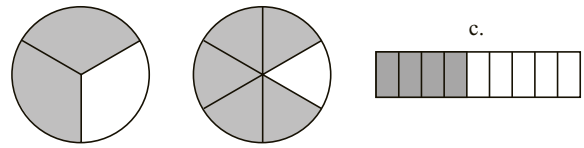
2. Divide \$18.00 by 4 to get the hourly wage:
   $18.00 \div 4 = 4.50$
   Then mulitply by 14 to calculate the weekend earnings:
   $4.50 \times 14 = 63.00$
   Sum the earnings to find the total $18 + 63 = 81$
   Mary earned \$81 altogether.

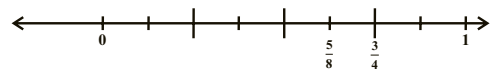
3. 3 \$5.00 shirts and 0 \$3.00 shirts
   2 \$5.00 shirts and 1 \$3.00 shirt
   1 \$5.00 shirt and 3 \$3.00 shirts
   0 \$5.00 shirts and 5 \$3.00 shirts

Section 3.1

Lesson 3.1.1 — Understanding Fractions

1. a. $\mathbf{\frac{1}{2}}$ b. $\mathbf{\frac{2}{5}}$ c. $\mathbf{\frac{5}{8}}$

2. a. b. c.
   
   Answers will vary, depending on the figure used.

3. 

   $\frac{3}{4}$ is further to the right, so $\frac{3}{4}$ is the greater number.

Lesson 3.1.2 — Improper Fractions

1. 8 sandwiches are divided between 5 people, so each gets $8 \div 5 = \mathbf{\frac{8}{5}}$ **sandwiches**.

2. \$63 is divided between 5 people, so each gets $\mathbf{\frac{63}{5}}$ **dollars**.

3. $\frac{11}{7} = \frac{7}{7} + \frac{4}{7} = 1 + \frac{4}{7} = \mathbf{1\frac{4}{7}}$.

Lesson 3.1.3 — More on Fractions

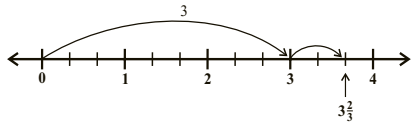
1. a. $23 \div 6 = 3$, remainder 5, so $-\frac{23}{6} = \mathbf{-3\frac{5}{6}}$

   b. $29 \div 6 = 4$, remainder 5, so $-\frac{29}{6} = \mathbf{-4\frac{5}{6}}$

2. a. $-6\frac{2}{5} = -\frac{6\times5+2}{5} = \mathbf{-\frac{32}{5}}$

   b. $-8\frac{5}{6} = -\frac{8\times6+5}{6} = \mathbf{-\frac{53}{6}}$

3. $\frac{11}{3} = 3\frac{2}{3}$, so...



Lesson 3.1.4 — Fractions and Decimals

1. a. It takes 10 dimes to make a dollar, so a dime is $\frac{1}{10}$ of a dollar. $\frac{1}{10}$ as a decimal is **0.1**.

   b. It takes 4 quarters to make a dollar, so a quarter is $\frac{1}{4}$ of a dollar. $\frac{1}{4}$ as a decimal is **0.25**.

   c. A half-dollar is $\frac{1}{2}$ of a dollar. $\frac{1}{2}$ as a decimal is **0.5**.

   d. It takes 20 nickels to make a dollar, so a nickel is $\frac{1}{20}$ of a dollar. $\frac{1}{20}$ as a decimal is **0.05**.

2. a. $\frac{2}{4} = 2 \times \frac{1}{4} = 2 \times 0.25 = \mathbf{0.5}$

   $\frac{3}{4} = 3 \times \frac{1}{4} = 3 \times 0.25 = \mathbf{0.75}$

   $\frac{4}{4} = 4 \times \frac{1}{4} = 4 \times 0.25 = \mathbf{1}$

   b. $\frac{2}{8} = 2 \times \frac{1}{8} = 2 \times 0.125 = \mathbf{0.25}$

   $\frac{5}{8} = 5 \times \frac{1}{8} = 5 \times 0.125 = \mathbf{0.625}$

   $\frac{10}{8} = 10 \times \frac{1}{8} = 10 \times 0.125 = \mathbf{1.25}$

   c. $\frac{3}{5} = 3 \times \frac{1}{5} = 3 \times 0.2 = \mathbf{0.6}$

   $\frac{5}{5} = 5 \times \frac{1}{5} = 5 \times 0.2 = \mathbf{1}$

   $\frac{10}{5} = 10 \times \frac{1}{5} = 10 \times 0.2 = \mathbf{2}$

Section 3.2

Lesson 3.2.1 — Multiplying Fractions by Integers

1. a. $4 \times \frac{2}{3} = \frac{4\times2}{3} = \mathbf{\frac{8}{3}}$

   b. $5 \times \frac{5}{6} = \frac{5\times5}{6} = \mathbf{\frac{25}{6}}$

   c. $4 \times \frac{2}{5} = \frac{4\times2}{5} = \mathbf{\frac{8}{5}}$

   d. $5 \times \frac{2}{3} = \frac{5\times2}{3} = \mathbf{\frac{10}{3}}$

2. a. negative
   b. positive
   c. negative

3. a. $6 \times \frac{4}{5} = \frac{6\times4}{5} = \frac{24}{5} = \mathbf{4\frac{4}{5}}$

   b. $-\frac{2}{7} \times 4 = -\frac{2\times4}{7} = -\frac{8}{7} = \mathbf{-1\frac{1}{7}}$

   c. $-7 \times \frac{3}{11} = -\frac{7\times3}{11} = -\frac{21}{11} = \mathbf{-1\frac{10}{11}}$

   d. $7 \times \frac{5}{6} = \frac{7\times5}{6} = \frac{35}{6} = \mathbf{5\frac{5}{6}}$

Lesson 3.2.2 — More on Multiplying Fractions by Integers

1. Mr. Wilson needs a bookshelf that can hold 12 of the $1\frac{1}{3}$-inch books, so it must be $12 \times 1\frac{1}{3} = 12 \times \frac{4}{3} = \frac{12 \times 4}{3} = \frac{48}{3} = \mathbf{16\ inches}$ wide.

2. Mrs. Martinez's fence will be $12 \times 3\frac{3}{5} = 12 \times \frac{18}{5} = \frac{12 \times 18}{5} = \frac{216}{5} = \mathbf{43\frac{1}{5}}$ **feet** long.

3. Connor's wall will be $7 \times 1\frac{3}{5} = 7 \times \frac{8}{5} = \frac{56}{5} = \mathbf{11\frac{1}{5}}$ **feet** high.

Lesson 3.2.3 — Multiplying Fractions by Fractions

1. a. $\frac{1}{3} \times \frac{1}{4} = \mathbf{\frac{1}{12}}$ b. $\frac{3}{5} \times \frac{1}{4} = \mathbf{\frac{3}{20}}$ c. $\frac{2}{3} \times \frac{2}{3} = \mathbf{\frac{4}{9}}$

2. $\frac{4}{7} \times \frac{5}{9} = \frac{4 \times 5}{7 \times 9} = \mathbf{\frac{20}{63}}$

3. a. $\frac{4}{3} \times \frac{2}{3} = \frac{4 \times 2}{3 \times 3} = \mathbf{\frac{8}{9}}$

   b. $\frac{5}{3} \times \frac{5}{7} = \frac{5 \times 5}{3 \times 7} = \mathbf{\frac{25}{21}}$

   c. $\frac{5}{8} \times \frac{11}{9} = \frac{5 \times 11}{8 \times 9} = \mathbf{\frac{55}{72}}$

   d. $\frac{2}{5} \times \frac{4}{7} = \frac{2 \times 4}{5 \times 7} = \mathbf{\frac{8}{35}}$

Section 3.3

Lesson 3.3.1 — Dividing by Fractions

1. a. $\mathbf{\frac{3}{2}}$ b. $\mathbf{\frac{11}{7}}$ c. $\mathbf{\frac{17}{4}}$

   d. $2\frac{2}{3}$ as an improper fraction is $\frac{8}{3}$, so its reciprocal is $\mathbf{\frac{3}{8}}$.

   e. $4\frac{6}{7}$ as an improper fraction is $\frac{34}{7}$, so its reciprocal is $\mathbf{\frac{7}{34}}$.

   f. $7\frac{2}{5}$ as an improper fraction is $\frac{37}{5}$, so its reciprocal is $\mathbf{\frac{5}{37}}$.

2. a. $\frac{3}{4} \times \frac{1}{5}$.

   b. $\frac{3}{4} \div 5 = \frac{3}{4} \times \frac{1}{5} = \mathbf{\frac{3}{20}}$

3. $\frac{3}{4} \div 2 = \frac{3}{4} \div \frac{2}{1} = \frac{3}{4} \times \frac{1}{2} = \mathbf{\frac{3}{8}}$

Lesson 3.3.2 — Solving Problems by Dividing Fractions

1. Each person could have $\frac{2}{3} \div 5 = \frac{2}{3} \times \frac{1}{5} = \mathbf{\frac{2}{15}}$ **of a bowl**.

2. You would use **division** to split a number into equal parts.

3. **No**, $\frac{3}{4}$ of 8 is the same as $\frac{3}{4} \times 8$.

Section 3.4

Lesson 3.4.1 — Making Equivalent Fractions

1. Answers may vary — students should write **any three fractions equivalent to** $\mathbf{\frac{2}{3}}$, for example: $\frac{4}{6}$, $\frac{6}{9}$, and $\frac{8}{12}$.

2. a. $\frac{2}{3} = \mathbf{\frac{8}{12}}$ b. $\frac{5}{7} = \mathbf{\frac{45}{63}}$

   c. $\frac{3}{8} = \mathbf{\frac{12}{32}}$ d. $\mathbf{\frac{3}{8}} = \frac{27}{72}$

   e. $\mathbf{\frac{7}{9}} = \frac{28}{36}$

3. Factors of 12: 1, 2, 3, 4, 6, 12
   Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
   So the common factors of 12 and 36 are **1, 2, 3, 4, 6, and 12**.

Lesson 3.4.2 — Finding the Simplest Form

1. $60 = 2 \times 2 \times 3 \times 5$
   $90 = 2 \times 3 \times 3 \times 5$

2. a. $12 = 2 \times 2 \times 3$ and $18 = 2 \times 3 \times 3$. Both numbers have factors of 2 and 3, so the greatest common divisor is $2 \times 3 = \mathbf{6}$.
   b. $12 = 2 \times 2 \times 3$ and $30 = 2 \times 3 \times 5$. Both numbers have factors of 2 and 3, so the greatest common divisor is $2 \times 3 = \mathbf{6}$.
   c. $30 = 2 \times 3 \times 5$ and $40 = 2 \times 2 \times 2 \times 5$. Both numbers have factors of 2 and 5, so the greatest common divisor is $2 \times 5 = \mathbf{10}$.
   d. $30 = 2 \times 3 \times 5$ and $60 = 2 \times 2 \times 3 \times 5$. Both numbers have factors of 2, 3, and 5, so the greatest common divisor is $2 \times 3 \times 5 = \mathbf{30}$.
   e. $40 = 2 \times 2 \times 2 \times 5$ and $56 = 2 \times 2 \times 2 \times 7$. Both numbers have factors of 2 (three times), so the greatest common divisor is $2 \times 2 \times 2 = \mathbf{8}$.
   f. $40 = 2 \times 2 \times 2 \times 5$ and $60 = 2 \times 2 \times 3 \times 5$. Both numbers have factors of 2 (twice) and 5, so the greatest common divisor is $2 \times 2 \times 5 = \mathbf{20}$.
   g. $12 = 2 \times 2 \times 3$ and $56 = 2 \times 2 \times 2 \times 7$. Both numbers have factors of 2 (twice), so the greatest common divisor is $2 \times 2 = \mathbf{4}$.

Lesson 3.4.3 — Fraction Sums

1. a. $\frac{5}{9} + \frac{7}{9} + \frac{2}{9} = \frac{5+7+2}{9} = \mathbf{\frac{14}{9}}$

   b. $\frac{12}{40} + \frac{15}{40} - \frac{8}{40} = \frac{12+15-8}{40} = \mathbf{\frac{19}{40}}$

   c. $\frac{50}{70} + \frac{21}{70} - \frac{24}{70} = \frac{50+21-24}{70} = \mathbf{\frac{47}{70}}$

2. To add fractions that have the same **denominators**, just add the **numerators** and keep the **denominators** the same.

3. a. $\frac{4}{9} + \frac{2}{9} = \frac{4+2}{9} = \frac{6}{9} = \mathbf{\frac{2}{3}}$

   b. $\frac{5}{12} + \frac{4}{12} = \frac{5+4}{12} = \frac{9}{12} = \mathbf{\frac{3}{4}}$

   c. $\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5} = \mathbf{1\frac{2}{5}}$

   d. $\frac{7}{8} + \frac{5}{8} = \frac{7+5}{8} = \frac{12}{8} = 1\frac{4}{8} = \mathbf{1\frac{1}{2}}$

Lesson 3.4.4 — Fractions with Different Denominators

1. Student answers may vary for all parts.
   a. Multiples of 4: 4, 8, 12, 16, 20, ...
   Multiples of 6: 6, 12, 18, 24, ...
   So a common multiple of 4 and 6 is, for example, **12**.
   b. Multiples of 8: 8, 16, 24, 32, 40, 48, ...
   Multiples of 12: 12, 24, 36, 48, ...
   So a common multiple of 8 and 12 is, for example, **48**.
   c. Multiples of 10: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, ...
   Multiples of 14: 14, 28, 42, 56, 70, ...
   So a common multiple of 10 and 14 is, for example, **70**.
   d. Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120, ...
   Multiples of 15: 15, 30, 45, 60, 75, 90, 105, 120, ...
   So a common multiple of 8 and 15 is, for example, **120**.

2. a. $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \mathbf{\frac{5}{6}}$

   b. $\frac{3}{5} + \frac{1}{10} = \frac{6}{10} + \frac{1}{10} = \mathbf{\frac{7}{10}}$

   c. $\frac{3}{4} - \frac{5}{8} = \frac{6}{8} - \frac{5}{8} = \mathbf{\frac{1}{8}}$

   d. $\frac{1}{3} - \frac{5}{6} = \frac{2}{6} - \frac{5}{6} = -\frac{3}{6} = \mathbf{-\frac{1}{2}}$

Lesson 3.4.5 — Least Common Multiples

1. a. 8, 16, 24, 32, 40, 48, 56, 64, 72, 80
   b. 12, 24, 36, 48, 60, 72, 84, 96, 108, 120
   c. 18, 36, 54, 72, 90, 108, 126, 144, 162, 180
   d. 20, 40, 60, 80, 100, 120, 140, 160, 180, 200
   e. 24, 48, 72, 96, 120, 144, 168, 192, 216, 240
   f. 30, 60, 90, 120, 150, 180, 210, 240, 270, 300
   g. 36, 72, 108, 144, 180, 216, 252, 288, 324, 360

2. a. Multiples of 8: 8, 16, 24, 32, 40, ...
   Multiples of 12: 12, 24, 36, 48, ...
   So the least common multiple of 8 and 12 is **24**.
   b. Multiples of 12: 12, 24, 36, 48, ...
   Multiples of 18: 18, 36, ...
   So the least common multiple of 12 and 18 is **36**.
   c. Multiples of 12: 12, 24, 36, 48, 60, ...
   Multiples of 20: 20, 40, 60, ...
   So the least common multiple of 12 and 20 is **60**.
   d. Multiples of 20: 20, 40, 60, 80, 100, 120, ...
   Multiples of 24: 24, 48, 72, 96, 120, ...
   So the least common multiple of 20 and 24 is **120**.
   e. Multiples of 30: 30, 60, 90, 120, 150, 180, ...
   Multiples of 36: 36, 72, 108, 144, 180, ...
   So the least common multiple of 30 and 36 is **180**.
   f. Multiples of 20: 20, 40, 60, 80, 100, ...
   Multiples of 30: 30, 60, ...
   So the least common multiple of 20 and 30 is **60**.
   g. Multiples of 18: 18, 36, 54, 72, 90, ...
   Multiples of 30: 30, 60, 90, ...
   So the least common multiple of 18 and 30 is **90**.

Lesson 3.4.6 — Mixed Numbers and Word Questions

1. Convert them into equivalent fractions with the same denominator, then compare the numerators.

   $\frac{3}{4} = \frac{9}{12}$ and $\frac{2}{3} = \frac{8}{12}$, so $\mathbf{\frac{3}{4}}$ **is greater**.

2. a. $7 + 2\frac{3}{4} = \frac{7}{1} + \frac{11}{4} = \frac{28}{4} + \frac{11}{4} = \frac{39}{4} = \mathbf{9\frac{3}{4}}$

   b. $5\frac{2}{3} - 3 = \frac{17}{3} - \frac{3}{1} = \frac{17}{3} - \frac{9}{3} = \frac{8}{3} = \mathbf{2\frac{2}{3}}$

   c. $3\frac{4}{5} + 5 = \frac{19}{5} + \frac{5}{1} = \frac{19}{5} + \frac{25}{5} = \frac{44}{5} = \mathbf{8\frac{4}{5}}$

   d. $7\frac{5}{9} - 9 = \frac{68}{9} - \frac{9}{1} = \frac{68}{9} - \frac{81}{9} = -\frac{13}{9} = \mathbf{-1\frac{4}{9}}$

   e. $5 - 2\frac{5}{6} = \frac{5}{1} - \frac{17}{6} = \frac{30}{6} - \frac{17}{6} = \frac{13}{6} = \mathbf{2\frac{1}{6}}$

Section 3.5

Lesson 3.5.1 — Fractions and Percents

1. a. 57% b. 21% c. 87% d. 9% e. 50% f. 63%

2. a. $47\% = \mathbf{\frac{47}{100}}$ b. $40\% = \frac{40}{100} = \mathbf{\frac{2}{5}}$

   c. $65\% = \frac{65}{100} = \mathbf{\frac{13}{20}}$ d. $129\% = \mathbf{\frac{129}{100}}$

   e. $287\% = \mathbf{\frac{287}{100}}$

   f. $33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{33\frac{1}{3}}{100} \times \frac{3}{3} = \frac{100}{300} = \mathbf{\frac{1}{3}}$

3. $\frac{3}{4} = \frac{75}{100} = \mathbf{75\%}$

Lesson 3.5.2 — Percents and Decimals

1. a. 39% = **0.39** b. 57% = **0.57**
   c. 47% = **0.47** d. 137% = **1.37**
   e. 245% = **2.45** f. 75% = **0.75**

2. a. 0.56 = **56%** b. 0.99 = **99%**
   c. 1.07 = **107%** d. 1.47 = **147%**
   f. 3.58 = **358%** f. 0.4 = 0.40 = **40%**

3. a. 20% b. 25% c. $33\frac{1}{3}\%$ d. 50% e. 75% f. 100%

Lesson 3.5.3 — Percents of Numbers

1. She should **multiply 27 and 0.80 together**.
2. a. 30% of 60 = 0.3 × 60 = **18**
   b. 45% of 80 = 0.45 × 80 = **36**
   c. 60% of 20 = 0.6 × 20 = **12**
   d. 130% of 70 = 1.3 × 70 = **91**
3. He needs to **multiply 20 by 6**, because 20 is 10% and he wants to work out 6 × 10% = 60%.

Lesson 3.5.4 — Circle Graphs and Percents

1. $P = 100 - 40 - 25 - 20 =$ **15**
2. 100 – 44 – 24 = 32
   32% of people said they preferred red.
3. 150 – 78 – 16 = 56
   56 people said they would choose Raul.

Lesson 3.5.5 — Percent Increase

1. a. 176 × 0.1 = 17.6, so 176 increased by 10% is 176 + 17.6 = **193.6**.
   b. 124 × 0.25 = 31, so 124 increased by 25% is 124 + 31 = **155**
   c. 254 × 0.84 = 213.36, so 254 increased by 84% is 254 + 213.36 = **467.36**
2. 18% of $21.45 is 3.861, so Nisha left a tip of **$3.86**.
3. Adam paid tax of 8.5% × $18 = 0.085 × 18 = $1.53, so in total he paid 18 + 1.53 = **$19.53** for the shirt.

Lesson 3.5.6 — Percent Decrease

1. a. The amount of the decrease is 48 × 25% = 48 × 0.25 = 12. So to decrease 48 by 25%, take away 12. That leaves 48 – 12 = **36**.
   b. Decrease is 0.9 × 120 = 108.
   So the decreased amount is 120 – 108 = **12**.
   c. Decrease is 0.48 × 190 = 91.2
   So the decreased amount is 190 – 91.2 = **98.8**.
   d. Decrease is 0.35 × 150 = 52.5
   So the decreased amount is 150 – 52.5 = **97.5**.
2. The discount is 0.4 × $35 = $14.
   So with the discount, the price is $35 – $14 = **$21**.
3. The discount is 0.3 × $70 = $21.
   So with the discount, the price is $70 – $21 = **$49**.

Lesson 3.5.7 — Simple Interest

1. I = amount of interest
   P = principal (the original amount borrowed or saved)
   r = yearly rate of interest, as a decimal or a fraction
   t = time in years
2. $P = \$1000$
   $r = 6\% = 0.06$
   $t = 1$
   So, $I = Prt = 1000 \times 0.06 \times 1 = \$60$.
   That means that the total investment after one year is worth 1000 + 60 = **$1060**.
3. $P = \$400$
   $r = 5.3\% = 0.053$
   $t = 3$
   So $I = Prt = 400 \times 0.053 \times 3 = \$63.60$.
   So in total William can withdraw 400 + 63.60 = **$463.60**.

Section 4.1

Lesson 4.1.1 — Ratios

1. a. There are 2 shaded squares and 3 unshaded squares so the ratio is **2 to 3**, **2:3**, or $\frac{2}{3}$.
   b. There are 3 unshaded squares and 5 squares in total so the ratio is **3 to 5**, **3:5**, $\frac{3}{5}$.
2. a. The shorter tree is 5ft and the taller tree is 8ft so the ratio is: **5 to 8**, **5:8**, **or** $\frac{5}{8}$
   b. The taller tree is 8ft and the shorter tree is 5ft so the ratio is: **8 to 5**, **8:5**, **or** $\frac{8}{5}$
3. a. Number of squares to number of objects is 3 squares to 7 objects or **3 to 7**, **3:7**, **or** $\frac{3}{7}$
   b. Number of circles to number of objects is 4 circles to 7 objects or **4 to 7**, **4:7**, **or** $\frac{4}{7}$
   c. Number of squares to number of circles is 3 squares to 4 circles or **3 to 4**, **3:4**, **or** $\frac{3}{4}$
   d. Number of circles to number of squares is 4 circles to 3 squares or **4 to 3**, **4:3, or** $\frac{4}{3}$

Lesson 4.1.2 — Equivalent Ratios

1. The answer to each question was reduced by dividing each number of the ratio by the GCD of the two numbers.
   a. **2 to 3** GCD = 2
   b. $\frac{2}{3}$ GCD = 4
   c. **3:5** GCD = 4
   d. **3 to 5** GCD = 5
   e. $\frac{1}{3}$ GCD = 100
   f. **3:4** GCD = 25
2. The ratio is 5 to 15 which reduces to **1 to 3** using the GCD of 5.

3. The ratio of girls (18) to boys (30 – 18 = 12) is $\frac{18}{12}$.

   Using a GCD of 6, this cancels to $\mathbf{\frac{3}{2}}$.

Lesson 4.1.3 — Proportions

1. a. **3 to 2**, **3:2** or $\mathbf{\frac{3}{2}}$

   b. There are a total of 5 × 3 = 15 girls, and 5 × 2 = 10 boys.

   Therefore the requested ratio is: **15 to 10**, **15:10** or $\mathbf{\frac{15}{10}}$

   c. $\mathbf{\frac{3}{2}=\frac{15}{10}}$

2. **Yes, the overall ratio of cups of tea to cups of coffee is in proportion with those poured at the first table.** This can be shown by the proportion $\frac{3}{5}=\frac{21}{35}$. Multiplying the value of both the numerator and denominator of the first fraction by 7, gives both the numerator and denominator of the second fraction.

3. **No**, $\mathbf{\frac{5}{6}=\frac{65}{78}}$ by multiplying both numerator and denominator by 13. (or $\frac{11}{13}=\frac{66}{78}$ by dividing both the numerator and denominator by 6)

Lesson 4.1.4 — Proportions and Cross-Multiplication

1. a. No, the ratios are **not equivalent**.
   $30 \times 9 = 270$
   $5 \times 56 = 280$
   b. Yes, the ratios are **equivalent**.
   $20 \times 7 = 140$
   $4 \times 35 = 140$
   c. No, the ratios are **not equivalent**.
   $9 \times 8 = 72$
   $3 \times 21 = 63$

2. a. $\frac{y}{36}=\frac{42}{54}$
   $54y = 42 \times 36$
   $y=\frac{42\times36}{54}=28$

   b. $\frac{4}{12}=\frac{11}{z}$
   $4z = 11 \times 12$
   $z=\frac{11\times12}{4}=33$

   c. $\frac{12}{20}=\frac{v}{35}$
   $12 \times 35 = 20v$
   $v=\frac{12\times35}{20}=21$

   d. $\frac{30}{u}=\frac{48}{88}$
   $30 \times 88 = 48u$
   $u=\frac{30\times88}{48}=55$

Section 4.2

Lesson 4.2.1 — Similarity

1. a. $\frac{5}{1.25}=\frac{8}{2}=\frac{10}{2.5}=\mathbf{4}$

   b. $\frac{5}{7.5}=\frac{8}{12}=\frac{10}{15}=\mathbf{\frac{2}{3}}$ **(or 0.66...)**

2. The ratio of sides in Shape A = $\frac{11.5}{12}=0.958...$

   The ratio of sides in Shape B = $\frac{23}{36}=0.638...$

   The ratio of sides in Shape C = $\frac{46}{48}=0.958...$

   Therefore **shapes A and C are similar**.

3. **True, they are all similar**. All sides of a particular square are the same length, and all angles within any square are equal. If a square is scaled up, the ratio of one side length to another remains the same, thus all squares are similar.

Lesson 4.2.2 — Proportions and Similarity

1. a. $\frac{4}{3}=\frac{12}{F}$
   $4F = 12 \times 3$
   $F=\frac{36}{4}$
   $\boldsymbol{F = 9}$ **cm**

   b. $\frac{15}{9}=\frac{10}{G}$
   $15G = 10 \times 9$
   $G=\frac{90}{15}$
   $\boldsymbol{G = 6}$

c. $\frac{3.2}{1.6} = \frac{H}{3.2}$

$3.2 \times 3.2 = 1.6 \times H$

$H = \frac{10.24}{1.6}$

$\boldsymbol{H = 6.4}$ **inches**

2. Solve the proportion $\frac{2}{3} = \frac{6}{GH}$ to determine that the side length GH of the larger shape is 9 cm. If each side of the large shape is 9 cm long, and there are 6 sides, then the perimeter is 9 × 6 = **54 cm**.

3. The ratio of corresponding sides of the triangles is 4 : 1, so sides of the smaller triangle are one-fourth the length of corresponding sides of the larger triangle. So one side of the smaller triangle is 8 cm ÷ 4 = **2 cm**, while the other is 14 cm ÷ 4 = **3.5 cm**.

Lesson 4.2.3 — Scale Drawings

1. a. 1 inch is 30 miles, so 17 inches is 17 × 30 = **510 miles**.
   b. Since it shows that 11 inches represents about 220 miles,

   $\frac{220 \text{ miles}}{11 \text{ inches}} = 20 \text{ miles/inch}$, so **each inch represents 20 miles**.
   c. Since 1 in. = 20 mi, a distance of 5.5 in. on the map would indicate a real distance of 5.5 × 20 = **110 miles**. An alternative approach would be to solve the proportion: $\frac{1}{20} = \frac{5.5}{x}$

2. a. 8 × 14 = **112 cm**
   b. 4.3 × 14 = **60.2 cm**

3. Since 1 in. = 50 ft, length of Titanic = 50 × 17.6 = 880
   So the Titanic was about **880 ft** long.

Section 4.3

Lesson 4.3.1 — Customary and Metric Units

1. **No, Terrence is not correct.** Converting a smaller unit to a larger unit, like inches to feet, requires **division by a conversion factor greater than 1** (here, by 12), not multiplication.
   Or you can **multiply by a conversion factor less than 1** (here, 1/12).

2. Multiplying by **100**

3. a. 1 mile = 1760 yards
   6 miles = 6 × 1760 yards
   = **10,560 yards**
   b. From the conversion chart:
   1 yard = 3 feet and 3 feet = 36 inches
   So, 1 yard = 36 inches
   5 yards = 5 × 36
   = **180 inches**

Lesson 4.3.2 — Conversions and Proportions

1. As 1 yd = 3 ft, then the proportion to be solved is $\frac{1}{3} = \frac{153}{f}$,
   So, $f = 3 \times 153$
   $= 459$
   So the fullback ran **459 ft**.

2. 1 mi = 1760 yd, and 1 yd = 3 ft.
   So 1 mi = 1760 × 3 = 5280 ft.
   Therefore, using $\frac{1}{4} = 0.25$, the proportion which needs to be solved is: $\frac{1}{5280} = \frac{0.25}{x}$
   $x = 0.25 \times 5280$
   $x =$ **1320 ft** before the construction

3. As 1 m = 100 cm, then the proportion to be solved is: $\frac{1}{100} = \frac{28}{x}$
   $x = 100 \times 28$
   $x =$ **2800 cm** is the height of the wave.

Lesson 4.3.3 — Converting Between Unit Systems

1. 8.5 × 1.09 = **9.265 yd**

2. 1 yard = 0.91 meters
   1 meter = 100 centimeter
   15 yards would be 0.91 × 15 = 13.65 meters
   13.65 × 100 = **1365 centimeters**

3. 1 mile is 1.61 kilometers
   0.85 × 1.61 = 1.3685 kilometers
   1 km = 1000 meters
   1.3685 × 1000 = 1368.5 meters
   1 m = 1000 mm
   1368.5 times 1000 = **1,368,500 millimeters**

Lesson 4.3.4 — Other Conversions

1. Answers will vary but most calculators require the buttons to be pressed in the following order:
   – the minus sign,
   – 4,
   – 9,
   – decimal point
   – 5
   Other calculators may require the minus sign button to be pressed last, instead of first.

2. a. C = (41 – 32) ÷ 1.8
   C = (9) ÷ 1.8
   C = 5
   41 ºF is equal to **5 ºC**
   b. C = (293 – 32) ÷ 1.8
   C = (261) ÷ 1.8
   C = 145
   293 ºF is equal to **145 ºC**

c. C = (0 – 32) ÷ 1.8
C = (–32) ÷ 1.8
C = –17.78 (2 d.p.)
0 ºF is equal to **–17.78 ºC**

d. C = (212 – 32) ÷ 1.8
C = (180) ÷ 1.8
C = 100
212 ºF is equal to **100 ºC**

e. C = (99.5 – 32) ÷ 1.8
C = (67.5) ÷ 1.8
C = 37.5
99.5 ºF is equal to **37.5 ºC**

f. C = (201.2 – 32) ÷ 1.8
C = (169.2) ÷ 1.8
C = 94
201.2 ºF is equal to **94 ºC**

Section 4.4

Lesson 4.4.1 — Rates

1. $\frac{40 \text{ books}}{8 \text{ hours}} = \frac{40 \text{ books} \div 8}{8 \text{ hours} \div 8} = \frac{5 \text{ books}}{1 \text{ hour}}$
Which is **5 books/hour.**

2. $\frac{17.25\ \$}{5 \text{ lb}} = \frac{17.25\ \$ \div 5}{5 \text{ lb} \div 5} = \frac{3.45\ \$}{1 \text{ lb}}$
Which is **3.45 $/lb** or $3.45/lb.

3. $\frac{36.72\ \$}{12 \text{ gal}} = \frac{36.72\ \$ \div 12}{12 \text{ gal} \div 12} = \frac{3.06\ \$}{1 \text{ gal}}$
Which is **3.06 $/lb** or $3.06/gal.

Lesson 4.4.2 — Using Rates

1. The number of yards he needs to buy.

2. First take $\frac{1}{2}$ of 24, which is 12. Then multiply 12 by 78 which is **936 points**.

3. a. Kelly needs to **divide $18.00 by 2** to determine the cost of one kite ($9)
b. **Multiply** the answer to part a. ($9) **by 28** ($252)

Lesson 4.4.3 — Finding Speed

1. 2520 km = speed × 28 hrs
speed = $\frac{2520}{28}$ = 90, so the duck flew at about **90 km/h**.

2. Firstly, convert the mixed fraction to an improper fraction:
$1\frac{2}{3} = \frac{5}{3}$ Then use this in the formula:

$30 \text{ miles} = \text{speed} \times \frac{5}{3} \text{ hours}$

$\text{speed} = \frac{30 \times 3}{5}$

$\text{speed} = \mathbf{18 \text{ miles/hour}}$

3. Firstly, convert the mixed fraction to an improper fraction:
$6\frac{1}{2} = \frac{13}{2}$
Then use this in the formula:

$3185 \text{ miles} = \text{speed} \times \frac{13}{2} \text{ hours}$

$\text{speed} = \frac{3185 \times 2}{13}$

$\text{speed} = \mathbf{490 \text{ miles/hour}}$

Lesson 4.4.4 — Finding Time and Distance

1. Distance = 50 mi/h × 4.5 h = **225 mi**.

2. Distance = 60 × (6 × 5) = 60 × 30 = 1800,
so the geese migrate about **1800 miles**.

3. $\text{Time} = \frac{400}{55} = 7.\overline{27}$ hours

So he drives for about $\mathbf{7\frac{1}{4}}$ hours per day.

Lesson 4.4.5 — Average Rates

1. Up: $\text{Time} = \frac{0.75}{1.5} = 0.5$ hours

Down: $\text{Time} = \frac{0.75}{5} = 0.15$ hours

$\text{Speed} = \frac{\text{total distance}}{\text{total time}} = \frac{0.75 + 0.75}{0.5 + 0.15} = \frac{1.5}{0.65} = 2.307... \frac{\text{miles}}{\text{hour}}$

The canoeist's average speed was **2.3 miles per hour** (1 d.p.)

2. Unloaded: $\text{Time} = \frac{500}{65} = 7.69...$ hours

Loaded: $\text{Time} = \frac{1000}{60} = 16.\overline{6}...$ hours

$\text{Speed} = \frac{\text{total distance}}{\text{total time}} = \frac{500 + 1000}{7.69.. + 16.\overline{6}} = \frac{1500}{24.358....} = 61.57... \frac{\text{miles}}{\text{hour}}$

So the truck's average speed was **61.6 miles per hour** (1 d.p.)

3. $\text{Time} = \frac{5}{25} = 0.2$ hours

$\text{Time} = \frac{10}{15} = 0.\overline{6}$ hours

$\text{Speed} = \frac{\text{total distance}}{\text{total time}} = \frac{5+10}{0.2+0.\overline{6}} = \frac{15}{0.8\overline{6}} = 17.307... \frac{\text{miles}}{\text{hour}}$

So the average speed of the fish was **17.3 miles per hour** (to 1 d.p.)

Section 5.1

Lesson 5.1.1 — Median and Mode

1. Put the numbers in order: {68, 71, 75, 79, 82, 87, 94}
   The data set contains 7 numbers, so the median is the 4th value, when the numbers are arranged in order. The 4th value is 79, so **79** is the median.
2. Put the numbers in order: {2, 4, 5, 7, 9, 10}
   There are six numbers in the data set, so the median lies midway between the third and fourth values. The third and fourth values are 5 and 7, so the median is **6**.
3. a. Red occurs three times in the list, and no other color occurs more than twice, so **red** is the mode.
   b. You can't put the values in the data set in order because they aren't numerical. So **you can't find a median for this data set**.

Lesson 5.1.2 — Mean and Range

1. Add the six values in the data set:
   15 + 18 + 22 + 56 + 41 + 28 = 180
   Divide the total by the number of values in the data set:
   180 ÷ 6 = 30
   So **30** is the mean.
2. Add Javier's scores:
   88% + 94% + 82% + 98% + 95% + 83% = 540%
   Divide by the number of tests: 540% ÷ 6 = 90%
   Javier will get an A, because his grade average is exactly 90%.
3. Add Kerry's earnings:
   $94 + $87 + $56 + $91 + $82 + $100 = $510.
   Divide by the number of Saturdays: $510 ÷ 6 = $85
   So Kerry earned an average of **$85** each Saturday.

Lesson 5.1.3 — Extreme Values

1. In **set A**, 100 is significantly larger than all the other elements, so is an outlier. In **set C**, 100 is significantly smaller than all the other elements, so is an outlier.
2. a. The outlier is **46.** It is significantly larger than all the other elements, and probably represents a bus or coach rather than a car.
   b. Add all the elements: 3 + 4 + 6 + 2 + 5 + 46 = 66
   Divide by the number of elements in the set: 66 ÷ 6 = 11
   So the mean with the outlier is **11**.
   c. Add the selected elements: 3 + 4 + 6 + 2 + 5 = 20
   Divide by the number of elements: 20 ÷ 5 = 4
   So the mean without the outlier is **4**.
3. a. Add Kathleen's quiz scores:
   1 + 8 + 9 + 9 + 10 + 9 + 10 + 8 = 64
   Divide by the number of quizzes: 64 ÷ 8 = 8
   So Kathleen's average quiz score is **8**.
   b. Add Kathleen's selected quiz scores:
   8 + 9 + 9 + 10 + 9 + 10 + 8 = 63
   Divide by the number of quiz scores: 63 ÷ 7 = 9
   So Kathleen's new average quiz score would be **9**.

Lesson 5.1.4 — Comparing Data Sets

1. a. The data set is not numerical, so it has no range (you can't put the values in order, so there is no highest or lowest value).
   b. The data set has no median (you can't put the values in order, so there is no middle value).
   c. The data set is not numerical, so it has no mean (since the values are not numbers, you can't sum them).
   d. The data set does have a mode. The mode is the value which occurs most often, and green is listed four times, while no other value color more than twice.
2. **No** — you can't be sure that the data sets are identical. It is possible for two data sets to have the same mean, median, mode, and range, but contain different values.
3. **Survey 1** is most likely to have been taken on Tuesday. On Tuesday afternoon it is likely that no-one of school age would have been in the park. On Saturday morning many children are likely to be there, so it is likely that a much greater range of ages will have been recorded.

Section 5.2

Lesson 5.2.1 — Including Additional Data: Mode, Median, and Range

1. a. 8 occurs three times in the data set, and no other number occurs more than once, so **8 is the mode**.
   Range = 10 – 4 **= 6**
   b. Now 8 and 9 both occur three times in the data set, and no other number occurs more than once, so **8 and 9 are the new modes**.
   New range = 12 – 4 **= 8**
2. a. Red and silver both occur two times in the data set, and no other color occurs more than once, so **red and silver** are the modes.
   b. If one silver car is removed, red becomes the only color that occurs more than once, so **red** is the new mode.

Lesson 5.2.2 — Including Additional Data: Mean

1. a. Add the values: 3 + 3 + 4 + 4 + 8 + 9 + 15 + 17 + 23 + 29 = 115
   Divide by the number of players: 115 ÷ 10 = **11.5 goals**
   b. Add the values:
   3 + 3 + 4 + 4 + 8 + 9 + 15 + 17 + 23 + 29 + 0 = 115
   Divide by the number of players: 115 ÷ 11 = **$10.\overline{45}$ goals**
2. a. Add the learner's values:
   73 + 77 + 85 + 78 + 80 + 90 + 75 + 70 = 628
   Divide by the number of learner swimmers:
   628 ÷ 8 = **78.5 seconds**
   b. Add the experienced swimmer's values:
   45 + 39 + 41 + 42 + 46 + 39 + 42 = 294
   Divide by the number of experienced swimmers:
   294 ÷ 7 = **42 seconds**
   c. Add everybody's values:
   73 + 77 + 85 + 78 + 80 + 90 + 75 + 70 + 45 + 39 + 41 + 42 + 46 + 39 + 42 = 922
   Divide by the number of swimmers:
   922 ÷ 15= **61.5 seconds (to 1 decimal place)**

Section 5.3

Lesson 5.3.1 — Analyzing Graphs

1. a. There are three marks above the value "4 brothers and sisters", so there are **three students** with exactly 4 brothers and sisters.
   b. Looking at the line plot, all those students that have 3, 4, 5, or 6 brothers and sisters come into the category of having at least 3 brothers and sisters. So you need to add the marks above all of these values: 5 + 3 + 0 + 1 = **9 students**.
   c. The mode is the most commonly occuring value in the data set. Of all the values on the graph, 2 has the most marks above it. So **2 brothers and sisters** is the mode.

Lesson 5.3.2 — Finding the Mean and Median from Graphs

1. Turn the plot back into a data set:
   {3, 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 8, 9}
   There are 23 values in the data set, so the median will be the twelfth value. The median is **size 5**.

Lesson 5.3.3 — Other Types of Graphs

1. a. More people own **Border Collies** — the segment of the graph representing Border Collies is bigger than the segment representing Boxer dogs.
   b. Boxer, Labrador Retriever, Border Collie.

Section 5.4

Lesson 5.4.1 — Using Samples

1. a. The population is all the candy bars made by the company.
   b. The sample is every fifth candy bar that they make.
2. a. This set contains an element that is not in the original set.
   b. This is the original set, so it represents the whole population, and not a sample of it.
   c. The median is not an element of the original set.

Lesson 5.4.2 — Convenience, Random, and Systematic Sampling

1. a. Convenience sampling
   b. Systematic sampling
   c. Convenience sampling
   d. Random sampling
2. a. Convenience sampling
   b. Cid stood in the children's section to take his survey. So the likelihood is that most of the people that pass him will be children. This will bias the results of his survey towards the younger ages.

Lesson 5.4.3 — Samples and Accuracy

1. a. Add all the values:
   3 + 8 + 7 + 9 + 10 + 4 + 6 + 11 + 5 + 9 + 7 + 6 + 5 + 4 + 8 + 6 + 9 + 5 + 11 + 3 = 136
   Divide by the number of members: 136 ÷ 20 = **6.8 baskets**
   b. Add the sample values: 3 + 5 + 7 + 9 + 11 = 35
   Divide by the number of sample values: 35 ÷ 5 = 7 baskets
   Subtract the population mean: 7 – 6.8 = **0.2 baskets**
   c. Add the sample values: 3 + 4 + 5 + 6 + 11 = 29
   Divide by the number of sample values: = 5.8 baskets
   Subtract the population mean: 5.8 – 6.8 = **–1 basket**
2. a. Put the values in order:
   {13, 15, 16, 16, 17, 18, 19, 19, 20, 20, 20, 21, 21, 22, 25}
   There are 15 values in the data set, so the median is the eighth value, which is **19 years**.
   b. Put the sample values in order: {15, 17, 18, 20, 22}
   There are 5 values in the sample, so the sample median is the third value, which is 18 years .
   Subtract the population median: 18 – 19 = **–1 year**
   c. Put the sample values in order: {19, 20, 22, 22, 25}
   There are 5 values in the sample, so the sample median is the third value, which is 22 years .
   Subtract the population median: 22 – 19 = **3 years**

Lesson 5.4.4 — Questionnaire Surveys

1. a. This would result in a biased sample. Since the survey is being conducted outside the art department, the people you stop would be most likely to be art students. So they would probably be more likely to support an art club.
   b. This is random sampling. It should produce an unbiased sample.

c. This would result in a biased sample. Students only volunteer to take part having been told what the survey is about. So those who already have a strong opinion on the issue are more likely to volunteer to answer.

2. Yes, this is a biased sample. Since the people surveyed were given the coffee for free, they are less likely to be critical than if they had had to pay for it themselves.

Section 5.5

Lesson 5.5.1 — Evaluating Claims

1. The segment representing summer sales is the largest segment of the circle graph, showing that the store sold more swimwear in summer than in spring, autumn, or winter. So the circle graph does support the manager's claim.

Lesson 5.5.2 — Evaluating Displays

1. Devin's claim is not valid. The break in the horizontal scale has been used to emphasize the difference in lengths of the bars. This makes the bar for the "1 visit" category look much longer than the others, even though there is actually less than 5% difference between all of the categories.

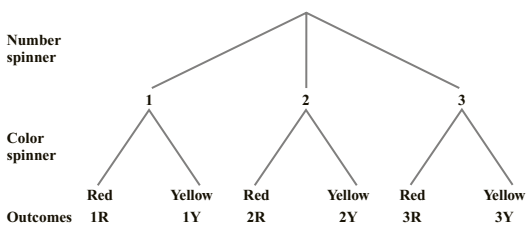
Section 6.1

Lesson 6.1.1 — Listing Possible Outcomes

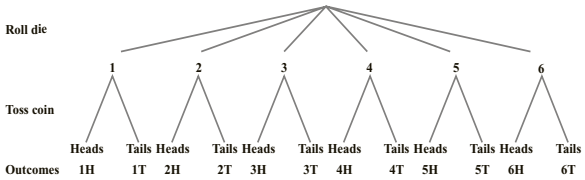
1. Red, yellow, blue and green
2. There are 2 outcomes for red, 2 for green, and 1 for yellow.
   So there are 5 possible outcomes.
3. Soccer, football, volleyball, basketball, and baseball
4. a. 2, 3, 5, 7
   b. 0, 1, 1, 4, 6
   c. 0, 1, 1, 2, 3
   d. There are no possible outcomes that match this event.
   e. 0, 1, 1, 2, 3, 4, 5, 6, 7
   f. There are no possible outcomes that match this event.

Lesson 6.1.2 — Tree Diagrams

1.



2.



Lesson 6.1.3 — Tables and Grids

1.

	Tennis shoes	Boots	Sandals
Green	GT	GB	GS
Red	RT	RB	RS
White	WT	WB	WS
Yellow	YT	YB	YS

2. $4 \times 4 = 16$
   Keris can create 16 different characters.

	Hair color A	B	C	D
Height 1	1A	1B	1C	1D
Height 2	2A	2B	2C	2D
Height 3	3A	3B	3C	3D
Height 4	4A	4B	4C	4D

3. Mrs. Jones must try $2 \times 3 = 6$ combinations:

	Soap A	Soap B
Bleach 1	1A	1B
Bleach 2	2A	2B
Bleach 3	3A	3B

Section 6.2

Lesson 6.2.1 — Probability

1. a. 35% = **0.35**
   b. 25% = **0.25**
   c. 5% = **0.05**

2. a. 1 out of the 10 sections contains a star,
   so the probability of spinning a star is $\frac{1}{10}$.
   b. 3 out of the 10 sections contain circles,
   so the probability of spinning a circle is $\frac{3}{10}$.

c. 4 out of the 10 sections contain squares,

so the probability of spinning a square is $\frac{4}{10} = \mathbf{\frac{2}{5}}$.

d. 2 out of the 10 sections contain triangles,

so the probability of spinning a triangle is $\frac{2}{10} = \mathbf{\frac{1}{5}}$.

Lesson 6.2.2 — Expressing Probability

1. $\frac{5}{8} = 5 \div 8 = \mathbf{0.625} = \mathbf{62.5\%}$

2. a. One quarter of the spinner is red, so the probability of spinning red is $\mathbf{\frac{1}{4}} = 1 \div 4 = \mathbf{0.25} = \mathbf{25\%}$.

   b. Half the spinner is green, so the probability of spinning green is $\mathbf{\frac{1}{2}} = 1 \div 2 = \mathbf{0.5} = \mathbf{50\%}$.

   c. One quarter of the spinner is yellow, so the probability of spinning yellow is $\mathbf{\frac{1}{4}} = 1 \div 4 = \mathbf{0.25} = \mathbf{25\%}$.

   d. None of the spinner is blue, so it is impossible to spin blue. The probability is **0** as a fraction and a decimal, and **0%** as a percent.

Lesson 6.2.3 — Counting Outcomes

1. a. Each of the two pairs of pants can be worn with each of the three shirts. $2 \times 3 = 6$, so Macauley can choose between **6 different outfits**.

   b. P(black pants) = $\frac{1}{2}$

   c. P(red shirt) = $\frac{1}{3}$

   d. P(grey pants with white shirt) = $\mathbf{\frac{1}{6}}$

2. a.

	Berry	Apple	Cherry
Plain	**PB**	**PA**	**PCh**
Ice cream	**IB**	**IA**	**ICh**
Custard	**CB**	**CA**	**CCh**

   b. 3 out of the 9 possible outcomes include berry pie,

   so P(berry pie) = $\frac{3}{9} = \mathbf{\frac{1}{3}}$

   c. 3 out of the 9 possible outcomes include custard,

   so P(custard) = $\frac{3}{9} = \mathbf{\frac{1}{3}}$

   d. 1 out of the 9 possible outcomes matches apple pie and ice cream, so P(apple pie and ice cream) = $\mathbf{\frac{1}{9}}$

Lesson 6.2.4 — Probability of an Event Not Happening

1. a. 5 out of 10 possible outcomes are squares,

   so P(square) = $\frac{5}{10} = \mathbf{\frac{1}{2}}$.

   b. 5 out of 10 possible outcomes are not squares,

   so P(not square) = $\frac{5}{10} = \mathbf{\frac{1}{2}}$.

   c. 2 out of 10 possible outcomes are circles,

   so P(circle) = $\frac{2}{10} = \mathbf{\frac{1}{5}}$.

   d. 8 out of 10 possible outcomes are not circles,

   so P(not circle) = $\frac{8}{10} = \mathbf{\frac{4}{5}}$.

   e. 3 out of 10 possible outcomes are triangles,

   so P(triangle) = $\mathbf{\frac{3}{10}}$.

   f. 7 out of 10 possible outcomes are not triangles,

   so P(not triangle) = $\mathbf{\frac{7}{10}}$.

2. a. $1 - \frac{3}{8} = \mathbf{\frac{5}{8}}$

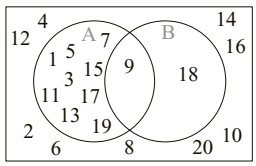
   b. $1 - \frac{2}{5} = \mathbf{\frac{3}{5}}$

   c. $1 - \frac{3}{4} = \mathbf{\frac{1}{4}}$

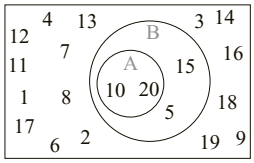
   d. 100% – 30% = **70%**

Lesson 6.2.5 — Venn Diagrams

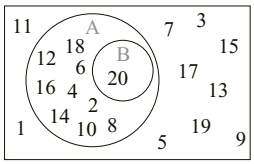
1. a.



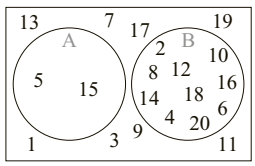
   b.



   c.



   d.



Lesson 6.2.6 — Combining Events

1. a. The numbers in the right-hand circle are multiples of 4. There are 7 multiples of 4 out of the 13 numbers on the diagram.

   P(multiple of 4) = $\mathbf{\frac{7}{13}}$

   b. The numbers in the left-hand circle are multiples of 10. There are 7 multiples of 10 out of the 13 numbers on the diagram.

   P (multiple of 10) = $\mathbf{\frac{7}{13}}$

   c. The numbers in both circles are multiples of 4 and 10. There are 3 multiples of 4 and 10 out of the 13 numbers on the diagram.

   P(multiple of 4 and a multiple of 10) = $\mathbf{\frac{3}{13}}$

   d. The numbers outside the circles are not multiples of 4 or 10. There are 2 numbers on the diagram outside the circles, which means there are 11 numbers inside one or both of the circles. 11 out of the 13 numbers on the diagram are multiples of 4 or 10.

   P(multiple of 4 or a multiple of 10) = $\mathbf{\frac{11}{13}}$

2. P (A or B) = P (A) + P (B) – **P(A and B)**

Section 6.3

Lesson 6.3.1 — Independent and Dependent Events

1. If the first cookie is oatmeal, there are more chocolate-chip cookies and fewer oatmeal cookies left to choose from on the second pick than if the first cookie is chocolate-chip. The chance of the second cookie being chocolate-chip is affected by the type of cookie chosen first, so the events are dependent.
2. No, because the names are replaced so the chance of any particular student being picked is the same every day.
3. No, because the outcome of the first spin does not affect any probabilities on the second spin.

Lesson 6.3.2 — Events and Probabilities

1. a. $\frac{3-1}{6-1} = \mathbf{\frac{2}{5}}$

   b. $\frac{3}{6-1} = \mathbf{\frac{3}{5}}$

   c. Yes, because the chance of Tryan getting a strawberry sweet is affected by whether or not Kasey gets a strawberry sweet.

2. If Ann's popsicle is cherry, the chance of Robert's popsicle being raspberry is $\frac{2}{5-1} = \frac{2}{4} = \frac{1}{2}$.

   However, if Ann's popsicle is not cherry, the chance of Robert's popsicle being raspberry is $\frac{2-1}{5-1} = \frac{1}{4}$.

   This shows that the chance of Robert getting a raspberry popsicle is affected by whether or not Ann gets a cherry popsicle, so the events are dependent.

Lesson 6.3.3 — Calculating Probabilities of Independent Events

1. a. P(1) = $\frac{3}{10} = \mathbf{0.3}$

   b. P(2) = $\frac{2}{10} = \mathbf{0.2}$

   c. P(3) = $\frac{1}{10} = \mathbf{0.1}$

   d. The tiles are replaced each time, making the events **independent**.

   e. Since the events are independent, you can multiply probabilities. P(1, 2, 3) = $0.3 \times 0.2 \times 0.1 = \mathbf{0.006}$

2. a. **Independent**, since the blocks are replaced after each pick.

   b. P(red) = $\frac{6}{18} = \frac{1}{3}$

   P(blue) = $\frac{8}{18} = \frac{4}{9}$

   Since the events are independent, you can multiply probabilities.

   P(red then blue) = $\frac{1}{3} \times \frac{4}{9} = \mathbf{\frac{4}{27}}$

   c. P(red) = $\frac{6}{18} = \frac{1}{3}$

   P(green) = $\frac{4}{18} = \frac{2}{9}$

   Since the events are independent, you can multiply probabilities. P(red then green) = $\frac{1}{3} \times \frac{2}{9} = \mathbf{\frac{2}{27}}$

   d. Since the events are independent, you can multiply probabilities.

   P(blue then red) = $\frac{1}{3} \times \frac{4}{9} = \mathbf{\frac{4}{27}}$

   Or you can use your answer to part b. above, since P(red then blue) = P(blue then red)

   e. P(green) = $\frac{4}{18} = \frac{2}{9}$

   Since the events are independent, you can multiply probabilities.

   P(green then green) = $\frac{2}{9} \times \frac{2}{9} = \mathbf{\frac{4}{81}}$

Section 6.4

Lesson 6.4.1 — Relative Frequency

1. 23 ÷ 169 = **0.136** (to 3 decimal places)

2.

Event	A	B	C	D
Frequency	764	327	643	266
Relative frequency	**0.382**	**0.1635**	**0.3215**	**0.133**

3. Monday: 3 ÷ 1000 = 0.003

   Tuesday: 8 ÷ 1000 = 0.008

   Wednesday: 8 ÷ 2000 = 0.004

   Thursday: 15 ÷ 2000 = 0.0075

   Friday: 20 ÷ 5000 = 0.004

   The parts were recalled on Tuesday and Thursday

Lesson 6.4.2 — Making Predictions

1. Relative frequency of sub-standard shirts = 15 ÷ 200 = 0.075, 3000 × 0.075 = 225
   Approximately 225 shirts are likely to be below standard.

2. Relative frequency of sales = 5 ÷ 100 = 0.05
   1500 × 0.05 = 75
   She is likely to make approximately 75 sales.

3. Relative frequency of sales = 1 ÷ 12 = 0.083
   Let n = the number of customers John will need to approach.
   $0.083n = 15$
   $n = 180$
   John is likely to approach approximately 180 customers before making 15 sales.

Section 7.1

Lesson 7.1.1 — Parts of a Circle

1. a. **NO** or **ON** — the diameter goes from one point on the circle, N to another point on the circle, O, through the circle's center.
   b. **LM**, **ML**, **NM**, **MN**, **MO**, or **OM** — the radius goes from the circle's center, M, to a point on the circle.

2. a. **SP**, **PS**, **SQ**, **QS**, **SR**, or **RS** — the radius goes from the circle's center, S, to a point on the circle.
   b. **QR** or **RQ** — the diameter goes from one point on the circle, Q to another point on the circle, R, through the circle's center.

3. a. **Yes**, because it goes from one point on the circle to a second point on the circle, through the center of the circle.
   b. **No**, a radius must start or end at the center of the circle with only one point on the circle.

Lesson 7.1.2 — Circumference and π

1. **The distance around a nickel** — a nickel is round, and the circumference is defined as the distance around a circle. A cube and a textbook are rectangles, and none of the Hawaiian islands are perfectly circular.

2. a. C = 3.14 × 10 = **31.4 inches**
   b. C = 3.14 × 14 = **43.96 cm**
   c. C = 3.14 × 8 = **25.12 ft**
   d. C = 3.14 × (6 × 2) = **37.68 yds**

3. $C = \pi d \approx \frac{22}{7} \times 28 =$ **88 inches**

Lesson 7.1.3 — Area of a Circle

1. **D** — Area of a circle = πr^2, where r = radius.
   The radius of the circle = half the diameter = 40 ÷ 2.

2. $A = \pi r^2 \approx 3.14 \times 15^2 =$ **706.5 ft²**

3. $A = \pi r^2 \approx \frac{22}{7} \times 7^2 = 22 \times 7 =$ **154 in²**

Section 7.2

Lesson 7.2.1 — Describing Angles

1. a. obtuse
   b. right
   c. obtuse
   d. acute
   e. right
   f. acute

2. a. right
   b. obtuse
   c. acute
   d. acute
   e. right
   f. straight

3. a. 35°, acute
   b. 105°, obtuse
   c. 90°, right
   d. 75°, acute
   e. 155°, obtuse

Lesson 7.2.2 — Pairs of Angles

1. a. adjacent
   b. adjacent
   c. adjacent
   d. not adjacent
   e. not adjacent — even though they have a common side, the two angles are not next to each other.
   f. not adjacent

2. ∠ABC and ∠FBE
∠CBD and ∠GBF
∠DBE and ∠ABG
∠CBE and ∠ABF
∠ABD and ∠GBE
∠CBG and ∠DBF
To determine the vertical angles, consider just two lines at a time. The "opposite" angles formed by these lines are vertical angles.

Lesson 7.2.3 — Supplementary Angles

1. The sum of the two supplementary angles equals 180°.
a. 180° – 45° = **135°**
b. 180° – 90° = **90°**
c. 180° – 125° = **55°**
d. 180° – 6° = **174°**

2. **∠A** and **∠D** are supplementary
∠B and **∠C** are supplementary.

3. a. ∠X + n = 37° + n = 180°, so $\boldsymbol{n}$ **= 180° – 37° = 143°**
b. ∠Y + n = 64° + n = 180°, so $\boldsymbol{n}$ **= 180° – 64° = 116°**
c. ∠Z + n = 108° + n = 180°, so $\boldsymbol{n}$ **= 180° – 108° = 72°**

Lesson 7.2.4 — The Triangle Sum

1. 70 + 80 = 150, 180 – 150 = 30, therefore ∠A = **30°**.

2. ∠F + 80° + 40° = 180°
∠F + 120° = 180°
∠F = 180° – 120°
∠F = **60°**

3. a. 180° – 90° – 44° = 180° – 134° = **46°**
b. 180° – 95° – 25° = 180° – 120° = **60°**
c. 180° – 34° – 66° = 180° – 100° = **80°**
d. 180° – 53° – 72° = 180° – 125° = **55°**

Lesson 7.2.5 — Complementary Angles

1. a. complementary
b. not complementary
c. complementary
d. not complementary

2. Each pair must sum to 90°.
a. 45°
b. 57°
c. 1°
d. 12°

Section 7.3

Lesson 7.3.1 — Classifying Triangles by Angles

1. **B** — an **obtuse** triangle
Since it has one obtuse angle it is an obtuse triangle.

2. A — right triangle
B — acute triangle
C — acute triangle
D — obtuse triangle

Lesson 7.3.2 — Classifying Triangles by Side Lengths

1. **B** — isosceles
It has two sides that have the same measure.

2. A — **scalene**, because none of the sides have the same length.
B — **equilateral**, because all three sides have the same length.
C — **isosceles**, because exactly two sides have the same length.

3. a. **Isosceles**, because exactly two of the sides have the same length.
b. **Scalene**, because none of the sides have the same length.
c. **Equilateral**, because all of the sides have the same length.

Lesson 7.3.3 — Types of Quadrilaterals

1. a. two
b. rectangle

2. a. square, rectangle, parallelogram
b. rhombus, parallelogram
c. trapezoid
d. rectangle, parallelogram
e. parallelogram

Lesson 7.3.4 — Drawing Quadrilaterals

1. Using a protractor, measure and draw a right angle. Extend each of the rays to the required lengths. Measure and draw another right angle at each of the free ends of the rays and extend these new rays until they meet.

Section 7.4

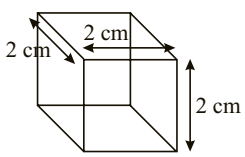
Lesson 7.4.1 — Three-Dimensional Figures

1. triangular prism

2. a. rectangular prism
b. cylinder
c. rectangular prism / cube

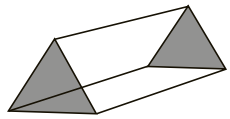
3. a. Faces — ABCD, EFGH, ABFE, ADHE, BCGF, DCGH
Edges — AE, AB, AD, BC, BF, CD, CG, DH, EF, EH, FG, GH
Vertices — A, B, C, D, E, F, G, H
b. **5 faces** — 3 on the sides and 2 on the top and bottom.
9 edges — 3 connecting the top to the bottom, 3 on the top, 3 on the bottom.
6 vertices — 3 on the top where edges meet and 3 on the bottom where edges meet.

Lesson 7.4.2 — Volumes of Rectangular Prisms

1. 

2. a. There are three sets of bases that could be picked for this rectangular prism:
Bases: ABCD and **EFGH**. **Height: AE** or **BF** or **CG** or **DH** or **8 cm**.
Bases: ABFE and **DCGH**. **Height: AD** or **BC** or **FG** or **EH** or **5 cm**.
Bases: EADH and **FBCG**. **Height: AB** or **DC** or **EF** or **GH** or **3 cm**. The bases are the top and bottom. The height is any edge going from one base to the other.
b. $V = Bh$, where V is the volume, B is the area of one base, h is the height. $V = 3 \times 5 \times 8 = 120$, so **$V = 120$ cm^3**.

Lesson 7.4.3 — Volumes of Triangular Prisms and Cylinders

1. 

2. Bases are **ABC** and **DEF**. The height is **AD** or **BE** or **CF** (the height would be any edge going between bases).

3. a. For a triangle, the area is calculated using the formula:
$A = \frac{1}{2}bh = \frac{1}{2} \times 15 \times 8 = 60$, so the area is **60 in^2**.
b. The volume of any prism = Bh. From a., you already know $B = 60$ in^2, so the calculation becomes:
$V = Bh = 60 \times 25 =$ **1500 in^3**.

Lesson 7.4.4 — Volumes of Compound Solids

1. a. The figure is made up of four triangular prisms, each with the same volume. The volume of one triangular prism is:
$V = Bh = \frac{1}{2} \times (16 \div 2) \times 7 \times 14 = 4 \times 7 \times 14 = 392$.
So the total volume of the figure = $392 \times 4 =$ **1568 cm^3**
b. The figure can be broken down into a rectangular prism and a cylinder.
Volume of the rectangular prism = $20 \times 12 \times 30$ = 7200 ft^3.
Volume of the cylinder = $\pi \times 24^2 \times 20$ $\approx 3.14 \times 576 \times 20 = 36{,}172.8$ ft^3.
So the total volume of the figure = $36{,}172.8 + 7200$ = **43,372.8 ft^3**.

2. The volume would be decreased by the volume of the triangular prism.

Section 7.5

Lesson 7.5.1 — Generalizing Results

1. n^2 is the rule for this pattern, so for the 10th figure there will be **100** 1 by 1 squares.

Lesson 7.5.2 — Proving Generalizations

1. a. **True** — both sides of an isosceles trapezoid meet each base at the same angle, and so must be the same length.
b. **False** — e.g. the top base could be larger than the bottom base and still be and isosceles trapezoid.